Quarks, Gluons, QCD

- Quarks: from a concept of classification to physics reality
- Deep inelastic electron scattering
  - Pointlike constituents: ‘partons’
  - Quantitative analysis: partons have spin $\frac{1}{2}$ and fractional charge
- $e^+e^-$ annihilation:
  - Number of quarks; color charge of quarks
  - Discovery of gluons
- QCD Lagrangian
  - Difference to QED
  - Quark-Gluon Plasma
Status of Quarks: ca 1966

- **Implausibility of Quark Model** (Jerome Friedmann (Nobel Prize 1976 for the experimental evidence for substructure)

- “...the idea that mesons and baryons are made primarily of quarks is hard to believe..” (M. Gell-Mann 1966)

- “Additional data are necessary and very welcome to destroy the picture of elementary constituents.” (J. Bjorken 1967)

- “I think Professor Bjorken and I constructed the sum rules in the hope of destroying the quark model.” (K. Gottfried 1967)

- “Of course the whole quark idea is ill founded.” (J.J. Kokkedee 1969)
Probing the size of the proton

• Probing the charge distribution, shown in figure

• Approach; measure the angular distribution of scattered electrons and compare to pointlike distribution

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(q)|^2 \quad \text{with} \quad q = k_i - k_f ; \quad |F(q)| \ldots \text{Form factor} \]

• Example: scattering of unpolarized electrons from static charge distribution \( Z e \rho(\vec{x}) \)

• For a static target: \( F(\vec{q}) = \int \rho(\vec{x}) e^{i\vec{q}\cdot\vec{x}} \, d^3x \) \ldots \text{Fourier transform of charge distribution} \]

• Form factor is Fourier transform of charge distribution

• Lorentz invariant four-momentum transfer

\[ q^2 = (E - E')^2 - (\vec{p} - \vec{p}')^2 \approx -4 \, E \, E' \, \sin^2 \left( \frac{\theta}{2} \right) \]
\[ e^- \mu^- \rightarrow e^- \mu^- \]

- Reaction is relevant for understanding lepton scattering on constituents
- Scattering cross section in Lab frame (muon at rest, mass \( M \))

\[
\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left( \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right)
\]

- Scattering cross section of electron on spin \( \frac{1}{2} \) particle
- Electron beam used to study dimension and internal structure of protons
Charge distribution of proton

- For $|q|$ small; (small energy transfer, large ‘equivalent’ wavelength of electron)
  \[ F(\vec{q}) = \int \left( 1 + i\vec{q} \cdot \vec{x} - \frac{(\vec{q} \cdot \vec{x})^2}{2} + \ldots \right) \rho(\vec{x}) \, d^3x = 1 - \frac{1}{6} |\vec{q}|^2 \langle r^2 \rangle \]
  assuming that charge distribution is spherically symmetric

- Low $|q|$, i.e. small angle scattering measures the mean square charge radius

- Cannot directly be applied to protons
  - Need to also consider magnetic moment; proton not static, will recoil

- Reference point-like cross-section is same as $e\mu$ scattering with $M_p$
  \[ \left( \frac{d\sigma}{d\Omega} \right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \theta / 2 \frac{E'}{E}} \left( A \cos^2 \frac{\theta}{2} - B \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right) \]
  where $A, B = 1$ for point-like proton; $E/E'$ from proton recoil
Charge distribution of proton

- Generalizing to extended source, one obtains two form factors (electric and magnetic) with $\kappa$ being the anomalous magnetic moment with the result
  \[
  A = \left( F_1^2 - \frac{\kappa^2}{4M^2} F_2^2 \right), \quad B = -\frac{q^2}{2M^2} \left( F_1 + \kappa F_2 \right)^2
  \]

- ‘Rosenbluth’ formula; the two form factors $F_{1,2}(q^2)$ summarize the structure of the proton; determined experimentally; formula reduces to pointlike formula for $\kappa=0$ and $F_1(q^2) = 1$

- In practice $G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2$, $G_M = F_1 + \kappa F_2$

- For protons: $\left< r^2 \right> = (0.81 \times 10^{-13} \text{ cm})^2$

- Nobel prize for Hofstaedter in 1961
Proton form factor versus $q^2$ 

Fourier transform of this Form factor is exponential 
Charge distribution 

$$\rho (r) = \rho_0 \exp(- \frac{q_0}{r})$$
Inelastic Electron-Proton scattering

• Probing the internal structure of the proton
  - Increase the momentum transfer $q^2$ of the photon, equivalent to photons of shorter wavelength
  - However, if proton is composite object, it will get excited, break up under large momentum transfer, producing system of particles with invariant mass $W$
  - For inelastic scattering:
    - Need two variables:
    - For example: $W$, $q^2$
The ep -> eX cross section

- The ep-> eX cross section as a function of the invariant mass of the particle system produced. The peak at invariant mass $W \approx M$ corresponds to scattering which does not breakup the proton; the peaks at higher $W$ correspond to excited states of the proton; beyond the resonances multiparticle states with large invariant mass result in a smooth behaviour.

(elastic peak at $W=M_p$ is reduced by factor 8.5)
Deep inelastic scattering

- Generalization of the inelastic scattering process follows the formalism for $e^- \mu^- \rightarrow e^- \mu^-$, but requires a more complicated description of the proton interaction, with two independent variables
  
  $Q^2 = -q^2$, $q$...four-momentum of virtual photon; $q = k-k'$;

  1. $Q$ defines scale $d$ probed: $d = \frac{hc}{Q} \approx 0.2 \text{ fm GeV}/Q \ldots 10^{-14}\text{ cm for } Q = 2 \text{ GeV}$
  2. $\nu = \frac{p \cdot q}{M}$ $M$..proton mass;

  or alternatively

  $x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M\nu}$ ...Bjorken scaling variable

  $y = \frac{p \cdot q}{p \cdot k}$...relative energy loss of electron in proton rest frame

  $W^2 = (p+q)^2 = M^2 + 2\nu M + q^2$ Invariant mass of final hadronic system

- Giving the final result

  $\left(\frac{d\sigma}{dE'd\Omega}\right)_{lab} = \frac{4\alpha^2 E'^2}{q^4} \left(W_2(\nu, q^2) \cos^2 \theta / 2 + 2W_1(\nu, q^2) \sin^2 \theta / 2\right)$

  with $W_1$ and $W_2$ to be determined experimentally... see later
Summary: electron scattering

- The differential cross section for $e\mu \rightarrow e\mu$, $ep \rightarrow ep$ (elastic) and $ep \rightarrow eX$ can be written as

\[
\left( \frac{d\sigma}{dE'd\Omega} \right)_{lab} = \frac{4\alpha^2 E'^2}{q^4} \{...\}
\]

- For $e\mu \rightarrow e\mu$

\[
\{...\} = \left( \cos^2 \theta / 2 - \frac{q^2}{2m^2} \sin^2 \theta / 2 \right) \delta \left( \nu + \frac{q^2}{2m} \right)
\]

- For $ep \rightarrow ep$ (elastic)

\[
\{...\} = \left( \frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \theta / 2 + 2\tau G_M^2 \sin^2 \theta / 2 \right) \delta \left( \nu + \frac{q^2}{2M} \right)
\]

- Integration over $\delta$-function gives

\[
\left( \frac{d\sigma}{d\Omega} \right) = \frac{\alpha^2}{4E^2 \sin^4 \theta / 2} \frac{E'}{E} \{...\}
\]

- For $ep \rightarrow eX$ very similar; see expression on previous slide
Studying the sub-structure of the proton

- The formalism developed for deep inelastic scattering $e p \rightarrow e X$ can be applied to the special case of probing a possible proton sub-structure
  - Using sufficiently small wavelength (i.e. sufficiently large $q^2$) it is possible to resolve a possible substructure, i.e. constituents
  - The breaking-up of the proton is described by the inelastic form factors $W_1$ and $W_2$
  - The scattering formalism is applied to electron scattering on the constituents, assuming certain properties
Probing the proton composition

- Assuming **pointlike** constituents (‘partons’) with **spin** $\frac{1}{2}$, the scattering cross section is related to $e\mu$-scattering with (for convenience $Q^2 = -q^2$)

$$2W_1^{po\text{int}}(\nu, Q^2) = \frac{Q^2}{2m^2} \delta (\nu - \frac{Q^2}{2m})$$

$$W_2^{po\text{int}}(\nu, Q^2) = \delta (\nu - \frac{Q^2}{2m})$$

- $m$ is the mass of the parton (or quark); pointlike: structureless Dirac particle
- Using the identity $\delta (x/a) = a \delta (x)$ one finds

$$2mW_1^{po\text{int}}(\nu, Q^2) = \frac{Q^2}{2m \nu} \delta (1 - \frac{Q^2}{2m \nu}) \quad \nu W_2^{po\text{int}}(\nu, Q^2) = \delta (1 - \frac{Q^2}{2m \nu})$$

- With the intriguing result that these functions depend only on the ratio $x = Q^2/2m\nu$ and not on $Q^2$ and $\nu$ independently -> Bjorken scaling
Probing the proton composition

- Summarizing and replacing the parton mass scale with the proton mass scale $M$

\[ MW_{1}^{point}(\nu, Q^{2}) \rightarrow F_{1}(\omega) \quad \nu W_{2}(\nu, Q^{2}) \rightarrow F_{2}(\omega) \]

for large $Q^{2}$ and $\omega = 2M\nu/Q^{2}$; at a given $\omega$, the structure functions are measured to be independent of $Q^{2}$ at constant Bjorken $x$
- Inelastic structure functions are independent on $Q^{2}$ -> constituents are pointlike and quasi-free (inside the proton)
- One experimental example
- Structure function = Fourier Transform of charge distribution \(\rightarrow\) St.F. is constant \(\rightarrow\) charge distrib. is pointlike!
What are the properties of the ‘partons’?

- Partons are spin ½, electrically charged pointlike particles

\[ E, p = \sum_i \int dx \, e_i^2 \]

- This picture recognizes that there are various partons in the proton: e.g. u, d quarks with different charges; uncharged gluons, with which the photon does not react; they carry different fraction \( x \) of the parent proton’s momentum and energy →

- Parton momentum distribution

\[ f_i(x) = \frac{dP_i}{dx} = \frac{p}{(1-x)p} \]
Parton momentum distribution functions

• $f_i(x)$ gives probability that parton $i$ carries fraction $x$ of the proton’s momentum $p$; all the fractions have to add up to 1

$$\sum_i \int dx \, x \, f_i(x) = 1$$

• Which leads to the following expressions for the structure functions

$$\nu W_2 (\nu, Q^2) \to F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$MW_1 (\nu, Q^2) \to F_1(x) = \frac{1}{2x} F_2(x)$$

with $x = 1/\omega = Q^2/2M\nu$, only dependent on $x$

• The momentum fraction is found to be identical to the kinematical variable $x$ of the virtual photon: the virtual photon must have the right value of $x$ to be absorbed by the parton with momentum fraction $x$
Looking at quarks inside the proton

- Proton is composed of the constituent quarks (u,d quarks) (or ‘valence’ quarks), gluons, and quark-antiquark pairs (‘sea’ quarks)

\[
\frac{1}{x} F_2^p (x) = \left( \frac{2}{3} \right)^2 [u^p (x) + \bar{u}^p (x)] + \left( \frac{1}{3} \right)^2 [d^p (x) + \bar{d}^p (x)] + \left( \frac{1}{3} \right)^2 [s^p (x) + \bar{s}^p (x)]
\]
Looking at quarks inside the proton

- Six unknown quark structure functions; additional information is provided by measuring electron-deuteron scattering, providing information on the corresponding neutron structure functions

\[ \frac{1}{x} F_2^n(x) = \left(\frac{2}{3}\right)^2 [u^n(x) + \bar{u}^n(x)] + \left(\frac{1}{3}\right)^2 [d^n(x) + \bar{d}^n(x)] + \left(\frac{1}{3}\right)^2 [s^n(x) + \bar{s}^n(x)] \]

- Due to isospin invariance their quark content is related
  - There are as many u quarks in the proton as d quarks in the neutron
    \[ u^p(x) = d^n(x) \equiv u(x) \quad d^p(x) = u^n(x) \equiv d(x) \quad s^p(x) = s^n(x) \equiv s(x) \]

- Additional constraints: quantum numbers of proton must be those of the uud combination
- Measurement of \( F_2(x) \) confirms charge assignment of the u and d quarks
Conceptual form of the structure functions

- If the Proton is a quark, then $F_2^P(x)$ is.
- Three valence quarks.
- Three bound valence quarks.
- Three bound valence quarks + some slow debris, e.g., $\gamma$ or $\pi$.

Small $x$
Valence quark distribution

From

\[
\frac{1}{x} \left[ F_2^p (x) - F_2^n (x) \right] = \frac{1}{3} \left[ u (x) - d (x) \right]
\]

one can directly measure the valence quark distributions
Quark structure functions

- From the analysis of deep inelastic scattering data
Quark structure functions: ‘state of the art’
Deep inelastic scattering (DIS) experiments at Stanford Linear Accelerator (SLAC)

- Developed in the late 1960’s; was at the time one of the largest experimental facilities
- Originally conceived to study elastic scattering -> extension to inelastic scattering met with some scepticism by the Program Committees: what can one learn?
- Established the quark structure
- Nobel prize (1990) for J.I. Friedman, H.W. Kendall and R.E. Taylor for ‘structure of the proton’
Summary: results from DIS

- From structure functions $F_2(x, Q^2) \approx F_2(x)$ -> nucleons are composed from pointlike constituents
- From $2x F_1(x) = F_2(x)$ -> constituents have spin $\frac{1}{2}$
- From experimental data on $F_2(x)$ for protons and neutrons (supplemented with data from DIS neutrino scattering) -> charge assignment for the u and d quarks
- From $\int F_2(x) \, dx$ -> quarks carry approximately 50 % of nucleon momentum; the rest is carried by the gluons; strong evidence for the reality and importance of gluons inside the nucleon
- Quantum numbers of the nucleon can be explained with the quantum number assignment of the quarks
e^+e^- Annihilations to Hadrons

- \( e^+ e^- \rightarrow Q \bar{Q} \rightarrow Q \rightarrow \text{hadron jet}, \, \bar{Q} \rightarrow \text{hadron jet} \)

- \( \sigma(e^+ e^- \rightarrow \text{anything}) \propto \frac{1}{s} \) (as for \( \mu^+ \mu^- \) production)
  - \( s \)…center of mass energy
- Peaks in cross-section are due to boson resonances
$e^+e^- \text{ Annihilations to Hadrons}$ vs center of mass energy $s$
Experimental proof of color charge of quarks

- Measurement of total cross section $\sigma (e^+e^- \rightarrow \text{hadrons})$ relative to $\sigma (e^+e^- \rightarrow \mu^+\mu^-)$
- Total cross section is obtained by summing over all contributing quarks:
  \[ \sigma(e^+e^- \rightarrow q\overline{q}) = \sigma(e^+e^- \rightarrow \mu^+\mu^-) \cdot N_C \sum_i q_i^2 \]
  - $N_C$ is the number of color charges (states)
  - The (three) color states of a quark have the same electric charge
  - The sum is over all energetically possibly produced quarks
- Measurement of the ratio
  \[ R = \sigma(e^+e^- \rightarrow q\overline{q}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-) = N_C \sum_i q_i^2 \]
  directly determines the number of color states
- Higher order effects (3 jets,..) modify $R$
  \[ R = R_0 (1 + \alpha_s(Q^2) / \pi + ...) \]
Measurement of $R$

\[ R = N_c \sum_f z_f^2 = N_c \cdot \left[ \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( -\frac{1}{3} \right)^2 \right] = N_c \cdot \frac{11}{9} \]
• Discovery of gluons in the observation of $e^+e^- \rightarrow$ three Jets → quark jet+ antiquark jet + gluon jet
  - gluon is radiated by a quark (or antiquark)
• Independent confirmation in proton-proton collisions (quark-gluon scattering) and DIS (electrons and neutrinos)
$e^+e^- \rightarrow$ three Jets

- Angular distribution of the gluon jet is sensitive to spin of gluon:
  - Spin of gluon = 1 (vector boson)
- Three-jet events can also be used to determine $\alpha_s$: rate of gluon radiation is proportional to $\alpha_s$
Summary: $e^+e^- \rightarrow \text{hadrons}$

- Measurements are consistent with
  - fractional charge for the quarks and three color states (value of $R$)
  - Quarks (Antiquarks) can radiate gluons -> gluons have similar reality as quarks
  - Gluon radiation can be quantitatively used to measure $\alpha_s$ and to determine the spin of the gluon ($S=1$)
Questions regarding Quarks:

- How big are quarks? (present limit: $< 10^{-17}$ cm)
- Are quarks composite systems?
- Why are their masses so different?
- What is the origin of flavor?
- Why are there six flavors?

- New tool for probing some of these questions, among any others: LHC: will probe distances $\sim 10$-18 cm
Theory of Strong Interactions: Quantum Chromodynamics (QCD)

- **Status (approx. 1970)**
  - Concept of quarks introduced for classification of hadrons
    - The ‘Eightfold Way’ by Gell-Mann; similar concept by Zweig
  - Classification needed another ingredient, ‘color’ charge of quarks
    - Required to avoid problem with Pauli exclusion principle →
    $$\Delta^{++} (uuu), \Omega^{-} (sss),...$$
  - Free quarks were not observed -> are quarks really particles ?
  - DIS showed that proton has a substructure -> partons
    - Detailed experiments confirmed partons to have the properties of quarks (fractional charge, spin $\frac{1}{2}$)
  - Quantum Electrodynamics (QED) confirmed with high experimental accuracy -> local gauge invariance as principle for deriving the Lagrangian of particle interactions
  - Experimental tests of nascent electroweak theory contemplated

- **Ingredients prepared for attacking the ‘hardest’ problem: Strong Interactions**
Color: Quantum Chromodynamics (QCD)

- Role of color
  - Example: ‘red’ quark carries one unit of ‘Redness’, zero greenness and blueness; antiquark carries *minus* one unit of redness
  - All naturally occurring particles are colorless
  - ‘Colorless’:
    - Total amount of each color is zero
      - or
    - All three colors are present in equal amounts
- Only colorless combinations are
  \[ q \bar{q} \quad (\text{mesons}), \quad qqq \quad (\text{baryons}), \quad \bar{q} \quad \bar{q} \quad \bar{q} \quad (\text{antibaryons}) \]
Quantum Chromodynamics (QCD)

- In QCD: color ‘charge’ is equivalent to electric charge in QED charge
- Fundamental vertex
  \[ q \rightarrow q + \text{gluon } g \]
- Analogous to \( e \rightarrow e + \gamma \)
- Bound state of \( q\bar{q} \)
- Scattering of two quarks
- Force between two quarks is mediated by the exchange of gluons
Quantum Chromodynamics QCD: similarities and differences to QED

- QED: one type of charge, i.e. *one* number (+, -); photon is neutral
- QCD: *three* kinds of color: red, green, blue
  - Fundamental process $q \rightarrow q + g$
    : color of quark (not its flavor may change in strong interactions)
    e.g.: blue up-quark $\Rightarrow$ red up-quark
    color is conserved $\Rightarrow$ gluon carries away the difference
    gluons are ‘bicolored’ with one positive and negative unit (e.g.: one unit of blueness and minus one unit of redness)
    $3 \times 3 = 9$ possibilities $\Rightarrow$ experimentally only 8 different gluons observed; ninth gluon would be ‘color singlet’ (color neutral) and therefore observable $\Rightarrow$ not observed, i.e. does not exist
QED and QCD: similarities and differences

- QED Lagrangian derived with the requirement of ‘local gauge invariance’; gauge group is U(1)
  \[ \mathcal{L} = \left[ i\hbar c \overline{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \overline{\psi} \psi \right] - \left[ \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} \right] - (q \overline{\psi} \gamma^\mu \psi) A_\mu \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]
  with \( A_\mu \) a new massless field such that \( A_\mu \rightarrow A_\mu + \partial_\mu \lambda \)

- QCD Lagrangian \( \mathcal{L} = i\hbar c \overline{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \overline{\psi} \psi \) for free quarks with \( \overline{\psi} = (\overline{\psi}_r \overline{\psi}_b \overline{\psi}_g) \), describes interaction of three (equal mass) color states-> require invariance under U(3), with U being a 3x3 matrix which can be written as \( U = e^{i\theta} e^{i\tilde{\lambda} \cdot \tilde{a}} \); \( \tilde{\lambda} \cdot \tilde{a} = \lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_8 a_8 \)

- Matrix \( e^{i\tilde{\lambda} \cdot \tilde{a}} \) has determinant 1-> belongs to SU(3) -> want to derive Lagrangian invariant under local SU(3) invariance

- \( \psi \rightarrow S\psi \), where \( S = e^{iq\tilde{\lambda} \cdot \Phi(x)/\hbar c} \); \( \Phi \equiv -(\hbar c / g_s)\tilde{a} \); \( g_s \) is coupling constant

- Complete QCD Lagrangian
  \[ \mathcal{L} = \left[ i\hbar c \overline{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \overline{\psi} \psi \right] - \left[ \frac{1}{4} G_{\mu\nu}^k G^{\mu\nu}_k \right] - g_s (\overline{\psi} \gamma^\mu \frac{\lambda^k}{2} \psi) G^k_\mu \]

1\textsuperscript{st} term: free quark; 2\textsuperscript{nd} term: gluon field; 3\textsuperscript{rd} term: quark-gluon interaction
QCD Lagrangian:

- Complete QCD Lagrangian

\[ \mathcal{L} = [i \hbar c \overline{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \overline{\psi} \psi] - \left[ \frac{1}{4} G_{\mu \nu}^k G_{k}^{\mu \nu} \right] - g_s (\overline{\psi} \gamma^\mu \frac{\lambda^k}{2} \psi) G_{\mu}^k \]

with

\[ G_{\mu \nu}^k = \partial_\mu G_{\nu}^k - \partial_\nu G_{\mu}^k - g_s f_{kjl} G_{\mu}^j G_{\nu}^l \]

- 1st part is analog to photon field in QED; 2nd term is new: quadratic in gluon field

\[ \mathcal{L}_{QCD} = (q\overline{q}) + g(q\overline{q}F) + (F^2) + g(F^3) + g^2(F^4) \]

- Lagrangian describes three equal-mass Dirac fields (the three colors of a given quark flavor) with eight massless vector fields (the gluons)
- Lagrangian applies to one specific quark flavor; need altogether six replicas of \( \psi \) for the six quark flavors
QCD: Gluon-Gluon coupling

Gluons, carrying color, (unlike the electrically neutral photon) may couple to other gluons ⇒ three and four gluon vertices ⇒ QCD more complicated (but also richer: allows far more possibilities)

- Coupling constant $\alpha_s \sim 1$ ⇒ higher order diagrams make significant (sometimes even even dominant) contributions: a real problem!
- However, triumph of QCD: discovery that $\alpha_s$ is NOT constant, but depends on the separation of the interacting particles ⇒ ‘running’ coupling constant:
  - $\alpha_s$ is large at large distances (larger than proton) ⇒ ‘confinement’
  - $\alpha_s$ is small at very short distances (smaller than proton) ⇒(‘asymptotic freedom’)
Detour to QED

- Also in electrodynamics: effective coupling also depends on distance
  - Charge q embedded in dielectric medium $\varepsilon$ (polarizable)
    - medium becomes polarized
    - Particle q acquires halo of negative particles, partially screening the charge q
    - at large distance charge is reduced to $q / \varepsilon$
    - in QED: vacuum behaves like dielectric
    - full of virtual positron-electron pairs
    - virtual electron attracted to q, positron repelled

- This vacuum polarization screens partially the charge at distances larger than $h/mc = 2.4 \times 10^{-10}$ cm (Compton wavelength of electron)
- Measurable, e.g. in structure of hydrogen levels
- NOTE: we measure the ‘screened’ charge, not the ‘bare’ charge
• QED: coupling constants modified by virtual effects (‘loop diagrams’) which ‘screens’ the electric charge and modifies the coupling constant as a function of the distance (or equivalently; of the momentum transfer of a reaction); observable: Lamb shift; anomalous magnetic moment

$$\alpha_{QED} = \frac{\alpha(0)}{1 - \left[\alpha(0)/3\pi\right] \ln \left|\frac{q^2}{(mc)^2}\right|} \quad \text{for } |q^2| >> (mc)^2$$

• Coupling constant $\alpha_{QED}$ varies only very weakly with $q^2$
QCD: More complicated

- Diagrams analogous to QED contribute to vacuum polarization

- $qqg$ vertex: contributes to increasing coupling strength at short distance (analogous to vacuum polarization in QED); strength depends on number of quark flavors $f$

- BUT: in addition: direct $gg$ vertex; strength depends on the number of gluons, i.e. number of colors $n$

- Competition between quark polarization diagrams, $\alpha_s \uparrow$ and gluon polarization, $\alpha_s \downarrow$ at short distances
QCD vacuum polarization and ‘Camouflage’

- In polarized medium the quark continuously emits and reabsorbs gluons, changing constantly its color
- Color-charged gluons propagate to appreciable distances, spreading the color charge of the quark, camouflaging the quark, which is source of the color charge
  - The smaller the region around the quark the smaller the effective color charge of the quark → color charge felt by quark of another color charge approaching the quark will diminish as the quark approaches the first one
- Net effect: competition between screening and camouflage
- **QCD: critical parameter** $a = 2f$ (of flavors) – $11n$ (number of colors)
  - if $a$ is positive (as in QED), coupling increases at short distance
  - in SM: $n = 3, f = 6; a = -21$; QCD coupling decreases at short distance
The screening effects happen in QCD (quark-antiquark loops), BUT in addition due to gluon couplings ‘camouflage’ of color charge. With the result for the coupling constant

$$\alpha_{QCD} = \alpha_s = \frac{\alpha_s(\mu^2)}{1 + \left[\alpha(\mu^2)/12\pi\right](11n - 2f)\ln\left[\frac{|q^2|}{\mu^2}\right]} \quad \text{for} \quad |q^2| >> \mu^2$$

With n= colors (3 in SM) and f= number of quark flavors (6 in SM); $\mu$ is a reference value around which $\alpha_s$ is evaluated.

At large $q^2 \alpha_s$ becomes less than 1-> perturbation theory is applicable; -> Asymptotic Freedom.

There are equivalent Feynman rules for quantitative calculations: perturbative QCD is quantitatively tested at the <1% level.
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Reflections by Frank Wilczek:
- We used a mixture of rigorous thinking and calculation together with wishful thinking and guesswork - whatever seemed to work!
- Experiment was crucial, both technically and psychologically. Also “experiment”.
- We worked very hard, but it didn’t seem laborious.

Lessons (Frank Wilczek)?
- Focus on paradoxes and surprising simplicities.
- It’s OK to make progress on one problem, without addressing all problems.)
- Good technique and hard work can be crucial to success.
QCD: One more difference

- quarks are confined in colorless packages
- experimental observations are indirect and are complicated manifestations of QCD
- force between two protons involves diagrams of the type shown
- reminiscent of the Yukawa $\pi$-exchange model

- QCD: theory must prove confinement: ongoing major task of theoretical research!
- QCD prediction at very high temperature (short range) phase transition to deconfined ‘Quark-Gluon Plasma’ -> subject of intense theoretical and experimental current research
Possible Scenario for Quark Confinement

- Concept for *proof* of quark confinement: potential energy increases without limit as quarks are pulled farther and farther apart -> energetically more favorable to produce quark-antiquark pairs
- Conclusive proof for confinement still lacking: long-range interaction difficult to treat theoretically
Quark-Gluon Plasma

- One ‘golden’ prediction of QCD is the ‘Quark-Gluon Plasma’, deconfined quarks and gluons at very high density or temperature
  - T ~ 170 MeV ~ 2*10^{12} K
  - thought to have been the primordial matter up to the first microsecond after the Big Bang
- Considered to be created in very energetic collisions of heavy nuclei (e.g. lead ions)
  - Was an active program at the CERN SPS; now actively being pursued at RHIC (Relativistic Heavy Ion Collider) at Brookhaven, USA
  - Major research activity at the LHC with one dedicated facility (ALICE... A Large Ion Collider Experiment)
At sufficiently high temperature Nuclear Matter undergoes a phase transition to deconfined quarks and gluons: Quark-Gluon Plasma;
Gravity: a fundamental correspondence

- Most-cited theoretical development of last decade
- Correspondence between anti-deSitter Gravity and conformal Quantum Field-Theories: AdS/CFT
  - Discovery within frame of Superstring-Theory
  - Strongly interacting Quantum Fieldtheorie (z.B. QCD) in 3 space and 1 time dimension → equivalently described with 5-dimensional Gravity theory
Applying the AdS/CFT correspondence:
Black Holes <-> Quark-Gluon Plasma

- Spectacular application: strongly coupled Quark-Gluon Plasma is described with the physics of black holes in 5 dimensions (and vice-versa)
- Successful prediction: viscosity of Quark-Gluon Plasma
- Quark-Gluon Plasma is a very active field of study at LHC