Symmetries and Conservation Laws

Symmetries and Conservation laws
Groups
Angular Momentum and extensions
Discrete Symmetries
  Parity, Charge Conjugation, Time Reversal
Violations of Symmetries
The CTP Theorem
Symmetries

- Symmetry: if a set of transformations, when applied to a system, leaves system unchanged →
  - transformation is a symmetry of system

- In classical physics: limited to applications in crystallography

- Einstein: speed of light is invariant under change of reference frame moving with uniform velocity: Invariance <-> symmetry

- Symmetries play an important role in particle physics, partly because they are related to conservation laws

- Understanding the origin of conservation laws guides the formulation of the quantitative description of the particle interactions: the inverse is also true: from symmetries of the interaction-> conservation laws

- Symmetry of crystals: shape is a ‘static’ symmetry

- Dynamical symmetries: associated with motion, interaction
Symmetries connected with Conservation Laws: Noether’s Theorem (1917)

• Every continuous symmetry of nature yields a Conservation Law
• Converse is also true: every conservation law reflects an underlying symmetry

<table>
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<th>Conservation Law</th>
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<td>Translation in space</td>
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</table>
Example of a symmetry: Translation Invariance

- Lagrangian of system with \( n \) degrees of freedom (could also use the Hamiltonian)
  - \( n \)-coordinates; \( n \)-velocities

\[
L = L(q_i, \dot{q}_i) \quad i = 1, 2, \ldots, n
\]

- Associated momenta, or momenta conjugate to the coordinates \( q_i \)

\[
p_i = \frac{\partial L}{\partial \dot{q}_i} 
\]

- Dynamical equations of motion

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad \Rightarrow \quad \frac{dp_i}{dt} = \frac{\partial L}{\partial q_i}
\]

- If Lagrangian of this system is independent of a particular coordinate \( q_m \)

\[
\frac{\partial L}{\partial q_m} = 0 \quad \Rightarrow \quad \frac{dp_m}{dt} = 0
\]

- Independent of a particular coordinate \( \Rightarrow \) translation invariant \( \Rightarrow \) conjugate momentum conserved
Symmetry: Definition

- **Symmetry:** is an operation, which – performed on a system – leaves system invariant, i.e. carries it into a configuration, which is indistinguishable from original one.
- **Example:** operation on equilateral triangle (‘Discrete Symmetry’)
  - unchanged under clockwise rotation of 120° ($R^+$)
  - unchanged under counter clockwise rotation ($R^-$)
  - unchanged under flip about vertical axis $a$ ($R_a$)
  - unchanged under flip about vertical axis $b$ ($R_b$)
  - unchanged under NO operation: identity ($I$)
  - unchanged under combined operation
  - Clockwise rotation under 240° ($R^+ R^+$) = $R^-$ …all possible symmetry operations defined by above operation
- **A circle has a continuous symmetry**
A group is a finite or infinite set of elements together with the group operation that satisfy the four fundamental properties:

- **Closure**: If $R_i$ and $R_j$ are in the set, then $R_i R_j$ (first perform $R_j$, then $R_i$) is also in the set.
  
  $$R_i \cdot R_j = R_k$$  
  * (closure is in German: ‘Geschlossenheit’)

- **Identity**: An element $I$ exists such that $I \cdot R_i = R_i I = R_i$ for all $R_i$.

- **Inverse**: For every element $R_i \rightarrow$ inverse, $R_i^{-1}$, exists, such that
  
  $$R_i \cdot R_i^{-1} = R_i^{-1} \cdot R_i = I$$

- **Associativity**: $R_i \cdot (R_j \cdot R_k) = (R_j \cdot R_i) \cdot R_k$
  - order in which the operations are performed does not matter as long as the sequence of the operands is not changed

- **Group Theory**: Allows systematic study of symmetries
Group Theory: Classification of symmetries

• Groups are the ‘Building Blocks’ of Symmetry

• Abelian Group: group elements commute: \( R_i \cdot R_j = R_j \cdot R_i \)
  - translation in space and time \( \Rightarrow \) Abelian Group

• Non-Abelian Group: \( R_i \cdot R_j \neq R_j \cdot R_i \)
  - rotations in three dimensions do not commute

• Finite Groups: example ‘Triangle’: has six elements

• Continuous Groups: e.g. rotations in a plane

• Discrete Groups: elements labelled by index that takes only integer values
Most Groups in Physics ⇒ represented by Groups of Matrices

• **In General:** every group \( G \) can be represented by a group of matrices: for every group element \( a \) ⇒ matrix \( M_a \)

• **Lorentz Group:** set of 4 x 4 matrices
  - transformation in 4-dimensional space

• **Unitary Groups U(n):** collection of all unitary, complex \( n \times n \) matrices
  - unitary matrix: inverse \( U^{-1} \equiv \text{transpose (complex) conjugate} \ U^* \)

• **Special Unitary Groups SU(n):** unitary matrices with determinant 1
  - Gell-Mann’s eightfold way corresponds to representations of SU(3)

• **Real Unitary Groups O(n):** orthogonal matrices: \( O^{-1} = O^\dagger \)

• **Real Orthogonal Groups SO(n):** determinant 1
  - SO(3): rotational symmetry of our world, related through Noether’s theorem to conservation of angular momentum,
Example 1: SO(3)

- Rotation in 3-dimensional space can be described with orthogonal, unimodular 3 x 3 matrices $R$

$$
\begin{pmatrix}
  x'
  \\
y'
  \\
z'
\end{pmatrix} = R
\begin{pmatrix}
  x
  \\
y
  \\
z
\end{pmatrix}
\quad R R^T = 1
$$

- Consider rotation about z-axis

$$
R_z = \begin{pmatrix}
  \cos \theta & \sin \theta & 0 \\
  -\sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
\end{pmatrix}
$$

- $R_z$ can also be presented as $R_z(\theta) = e^{i \theta J_z}$

$$
\frac{\partial R}{\partial \theta} \bigg|_{\theta = 0} = i J_z
$$

- $J_z$ are ‘Generators’ of the group $R_z$

with

$$
J_z = \begin{pmatrix}
  0 & -i & 0 \\
  i & 0 & 0 \\
  0 & 0 & 0
\end{pmatrix}
$$

, similarly for $J_x$, $J_y$
Example: SU(2)

- **SU(2):** complex, unitary, unimodular 2 x 2 matrices

\[
U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}
\] with \(a, b, c, d\) .... complex

- In general: 8 parameters; however
  unimodular: \(\det U = 1\); unitary: \(U^*U = 1\)

\[
U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}
\] with \(|a|^2 + |b|^2 = 1\) ; 3 free parameters

- \(U(\vec{\theta}) = e^{iT(\vec{\theta})} \ldots \vec{\theta}\) \Rightarrow elements of group; 3 Hermitian, lin. independent traceless matrices

- \(U = e^{i\frac{\theta_i}{2} \sigma_i}\) \Rightarrow Pauli Matrices, transformation in spinor space

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
Example for SU(2): Spin $\frac{1}{2}$

Spin $\frac{1}{2}$: most important spin system: proton, neutron, electron, quarks

- particle with $S = \frac{1}{2}$: $m_S = \frac{1}{2}$ (‘spin up’), $m_S = -\frac{1}{2}$ (‘spin down’)
- states can be presented by arrow: $\uparrow$; $\downarrow$
- better notation: two-component column vector, or spinor
  \[
  \begin{pmatrix}
  \alpha \\
  \beta
  \end{pmatrix}
  \]

- most general case of spin $\frac{1}{2}$ particle is linear combination
  \[
  \begin{pmatrix}
  \alpha \\
  \beta
  \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad \alpha, \beta \text{ complex numbers}
  \]

- measurement of $s_Z$ can only return value of $+\frac{1}{2} \hbar$ or $-\frac{1}{2} \hbar$

- $|\alpha|^2$ is probability that measurement of $s_Z$ yields $+\frac{1}{2} \hbar$

- $|\beta|^2$ is probability that measurement of $s_Z$ yields $-\frac{1}{2} \hbar$

Spin $\frac{1}{2}$ transforms according to 2-dim. representation of SU(2)
Flavour Symmetries

- Heisenberg, 1932: observed that proton and neutron (apart from charge) are almost identical
  - $M_p = 938.28 \text{ MeV}/c^2$; $M_n = 939.57 \text{ MeV}/c^2$
  - proposed to regard proton and neutron as two ‘states’ of the same particle, the ‘nucleon’
  - nucleon written as a two-component spinor
    $$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
  - by analogy to spin $\Rightarrow$ ‘Isospin’
  - Nucleon carries isospin $\frac{1}{2}$
  - $I$ is vector not in ordinary space, but in abstract ‘isospin space’, with $I_1, I_2, I_3$
  - Machinery developed for Angular Momenta can be applied
Concept of Isospin

- Heisenberg’s proposition: strong interactions are invariant under solution in isospin space (analog to: electrical forces are invariant under rotation in ordinary configuration space)

- Isospin invariance is ‘Internal Symmetry’ of system

- Noether’s theorem
  strong interactions is invariant under rotation in isospin space ⇒ isospin is conserved in all strong interactions

- Isospin assignment follows isospin assignment of the quarks
  - $u$ and $d$ form doublet: $u = \left| \frac{1}{2} \frac{1}{2} \right>$; $d = \left| \frac{1}{2} - \frac{1}{2} \right>$
  - all other quarks carry isospin zero

- Isospin formalism associated with SU(2)
  - strong interactions are invariant under internal symmetry group SU(2)
Dynamical Consequences of Isospin

- Two-nucleon system ($l_3$...is additive quantum number, as is spin)
  - symmetric isotriplet: $|11\rangle = pp; |10\rangle = \left(\frac{1}{\sqrt{2}}\right)(pn + np); |1 - 1\rangle = nn$
  - antisymmetric isosinglet: $|00\rangle = \left(\frac{1}{\sqrt{2}}\right)(pn - np)$ is the deuteron (if it were in triplet, all three states would have to occur)
  - Isosinglet state is totally insensitive to rotation in isospin space

- Nucleon-nucleon scattering
  - $p + p \rightarrow d + \pi^+; p + n \rightarrow d + \pi^0; n + n \rightarrow d + \pi^-$
  - deuteron $l = 0$; isospin states on right side are $|11\rangle, |10\rangle, |1 - 1\rangle$,
    on left side $pp = |11\rangle; nn = |1 - 1\rangle, pn = \left(\frac{1}{\sqrt{2}}\right)(|10\rangle + |00\rangle)$
  - only $l = 1$ combination contributes (final state is pure $l = 1$)
    - scattering amplitude in ratio $1 : \frac{1}{\sqrt{2}} : 1$ and cross sections (square of amplitudes) $1 : \frac{1}{2} : 1$, consistent with measurement
Dynamical Consequences of Isospin: Δ(1232)

- Four members of the Δ(1232) family, mass 1232 ±1 MeV/c², I=2/3, corresponding to four charged states, four I₃ projections of I=3/2 multiplet

- If isospin is good symmetry ⇒ total decay rates must be identical

<table>
<thead>
<tr>
<th>Charge State of Δ</th>
<th>I₃</th>
<th>Final State</th>
<th>Expected Rate</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ⁺⁺ (uuu)</td>
<td>3/2</td>
<td>pπ⁺</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Δ⁺ (uud)</td>
<td>1/2</td>
<td>pπ⁰</td>
<td>x</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>nπ⁺</td>
<td>1 – x</td>
<td>1/3</td>
</tr>
<tr>
<td>Δ⁰ (udd)</td>
<td>−1/2</td>
<td>pπ⁻</td>
<td>y</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>nπ⁰</td>
<td>1 – y</td>
<td>2/3</td>
</tr>
<tr>
<td>Δ⁻ (ddd)</td>
<td>−3/2</td>
<td>nπ⁻</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Ratio for decays involving a p or n in final state are the same
  \[1 + x + y = 1 – x + 1 – y + 1\]

- Ratio for decays involving π⁺, π⁰ or π⁻ are the same
  \[1 + 1 – x = x + 1 – y = y + 1\]

- Unique set of consistent solution shown in table, in agreement with measurements
Extension of Isospin Concept

• Eight baryons have approximately equal mass
  - tempting to regard them as a supermultiplet, belonging to the same representation of an enlarged symmetry group, in which SU(2) of isospin would be a subgroup

• Gell-Mann found that the corresponding symmetry group is SU(3)
  - octets are represented by the 8-dimensional representation of SU(3), i.e. the baryons belonging to this octet are transformed by the 8-dimensional representation of SU(3)
  - decuplets are represented by the 10-dimensional representation of SU(3)
Extension of Isospin Concept

- Difficulty: no particles fall into the fundamental, 3-dimensional representation (in contrast to nucleon) ⇒ emerging idea of quarks

- Gell-Mann: quarks: $u, d, s$ transformed according to the 3-dimensional representation of SU(3) which breaks down into isodoublet $(u,d)$ and isosinglet $(s)$ under SU(2)

- In this concept baryons, consisting of three quarks, $qqq...R(3) \otimes R(3) \otimes R(3) = 3 \otimes 3 \otimes 3 = 1 + 8 + 8 + 10$ decomposed into irreducible representations, with which the octets and decuplets can be identified
Caveat in this Hierarchy

• **Isospin, SU(2), symmetry is very good symmetry; mass of members of isospin multiplets differ by only few %**

• **Discrepancies become very large, when including the heavier quarks in this concept ⇒ can be traced to the bare quark masses**

<table>
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<tr>
<th>Quark Flavour</th>
<th>Base Mass</th>
<th>Effective Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>2</td>
<td>336</td>
</tr>
<tr>
<td>d</td>
<td>5</td>
<td>340</td>
</tr>
<tr>
<td>s</td>
<td>95</td>
<td>486</td>
</tr>
<tr>
<td>c</td>
<td>1300</td>
<td>1550</td>
</tr>
<tr>
<td>b</td>
<td>4200</td>
<td>4730</td>
</tr>
<tr>
<td>t</td>
<td>174000</td>
<td>177000</td>
</tr>
</tbody>
</table>

• **Effective mass is consequence of strong interactions due to confinement inside hadrons**
  - effective masses of *u, d, s* almost equal
  - Effective masses of *c, b, t* are very different (bare masses are different)

• **We have no fundamental explanation for the value of the bare quark masses**
Discrete symmetry: symmetry that describes non-continuous changes

Fundamental symmetry operations in particle physics:

- parity transformation (spatial inversion $P$)
- particle-antiparticle conjugation (charge conjugation $C$)
- time inversion ($T$)

According to the kind of interaction, the result of such a transformation may describe a physical state occurring with the same probability ("the symmetry is conserved") or not ("the symmetry is broken" or "violated").
Discrete Symmetries: Parity

- ‘Reflection’: arbitrary choice of mirror plane → better to consider
- Inversion = reflection, followed by 180° rotation
- Right handed coord. system → left handed coord. system
- $P$ ... parity operator, denoting inversion
  - applied to vector $a \ P(a) = -a$ (‘polar’ vector) (vector in opposite direction)
  - applied to cross product $c = a \times b \ P(c) = c$ (‘axial’ vector)
    (magnetic field; angular momentum are axial vectors)

\[
\begin{pmatrix}
ct \\
x \\
y \\
z
\end{pmatrix}
\rightarrow
\begin{pmatrix}
ct \\
-x \\
-y \\
-z
\end{pmatrix}
\]

inversion: every point is carried through origin to diametrically opposite location
Parity

- $P (a \cdot b) = (a \cdot b)$ scalar
- $P (a \cdot [b \times c]) = - a [b \times c]$ pseudoscalar
- $P^2 = I$; parity group has two elements: $I, P$
  - eigenvalues of $P$ are $+1, -1$
    - scalar: $P(s) = s \quad P = +1$
    - Pseudoscalar: $P(p) = -p \quad P = -1$
    - Vector (polar vector): $P(v) = -v \quad P = -1$
    - Pseudovector (axial vector): $P(a) = a \quad P = +1$
    - Angular momentum $\vec{L} = \vec{r} \times \vec{p} \rightarrow P \rightarrow - \vec{r} \times - \vec{p} = \vec{L}$

- Hadrons are eigenstates of $P$ and can be classified according to their eigenvalue
  - $P$ (fermion) = $-P$ (antifermion) $P$ (quark) = $+1$, $P$ (antiquark) = $-1$
  - $P$ (photon) = $-1$ ($S=1$; vector particle, represented by vector potential)
  - $P$ is multiplicative
  - $P$ (composite system in ground state) = $P$ (of product)
  - Baryon Octett $P = (1)^3$; meson nonett $P = (-1) ( +1 ) = -1$
Discrete Symmetries: Parity

• Transformation on state may describe a physical state occurring with
  - the same probability -> ‘symmetry is conserved’
  - or not: ‘symmetry is not conserved’, ‘is broken’, ‘is violated’
• Prior to 1956: ‘obvious’ that laws of physics are ambidextrous: the
  mirror image of a physical process is also a perfectly possible process
• Mirror symmetry (‘Parity Invariance’) considered to be ‘self-evident’.
• Lee and Yang: what are the experimental proofs of this assumption?
  - ample evidence for parity invariance in electromagnetic and strong
    processes
  - NO evidence in weak interactions
  - proposed experiment to settle the question -> result: Parity is
    violated in weak interaction processes!
• Nobel Prize for Lee and Yang in 1957
Observation of Parity Violation by Wu and Collaborators (1957)

- Study decay of $^{60}Co \rightarrow ^{60}Ni + e^- + \bar{\nu}_e$

- polarized matter
  - $^{60}Co$ at 0.01 Kelvin inside solenoid
    - high proportion of nuclei aligned
  - $^{60}Co$ (J=5) → $^{60}Ni^*$ (J=4) (similar to $\beta$-decay)
    - electron spin $\sigma$ points in direction of $^{60}Co$ spin $J$
    - conservation of angular momentum
    - degree of $^{60}Co$ alignment determined from observation of $^{60}Ni^*$ $\gamma$-rays

- observed electron intensity:

$$I(\vartheta) = 1 - \left( \frac{\sigma \cdot p}{E} \right) = 1 - \frac{v}{c} \cos \vartheta$$

- $\vartheta$: angle between electron ($p$) and spin ($J$)

- If Parity were conserved, would expect electrons equally frequently emitted in both directions

- Under Parity, $p$ changes sign, but not Spin → expectation value of angular distribution changes sign – if right- and left-handed coordinate system are equivalent → only possible, if distribution is uniform → Observed distribution not uniform → Parity is violated

M. Jeitler
Parity violation

If right handed and left handed C.S. are physically equivalent -> must observe an identical value in the two frames

If parity is conserved:

$$\langle \cos \vartheta \rangle = \frac{\langle \vec{s} \cdot \vec{p} \rangle}{|\vec{s}||\vec{p}|} = P \left( \frac{\langle \vec{s} \cdot (-\vec{p}) \rangle}{|\vec{s}||\vec{p}|} \right) = -\langle \cos \vartheta \rangle$$

Electrons must be emitted with equal probability in both directions
Wu-Ambler et al Experiment

Detail of apparatus

Published results
Chen Ning Yang and Tsung-Dao Lee
(Nobel prize 1957)
Parity Violation in Weak Interaction

- Parity violation is feature of weak interaction
  - is ‘maximally’ violated
  - most dramatically revealed in the behaviour of neutrinos

- ‘Helicity’ of neutrinos
  - particles with spin s travelling with velocity v along z-axis
  - Sign of projection of the spin vector on momentum vector = helicity of particle
  - particle with spin ½ can have
    - helicity 1 (m_s = ½) (‘right-handed’)
    - helicity -1 (m_s = -½) (‘left-handed’)

- Helicity of massive particle is NOT Lorentz-invariant
- Helicity of massless particle, travelling with v=c, is Lorentz-invariant
Helicity of Neutrinos

- Photons can have helicity +1 or -1, representing left and right circular polarization (feature of Lorentz invariance)
- Neutrinos are found to be always left-handed (helicity $H = -1$)
  Antineutrinos are found to be always right-handed (helicity $H = 1$)
- Parity violation in weak interaction is a consequence of this fact
  - Mirror image of neutrino does not exist
- Observation of neutrino helicity: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
  - Pion at rest $\Rightarrow \mu$ and $\bar{\nu}$ energy back-to-back; spin of pion $s=0 \Rightarrow$ spin of muon and antineutrino must be oppositely aligned
  - $\mu^-$ muon is always observed to be right-handed $\Rightarrow \bar{\nu}$ must be right-handed
Helicity of Neutrino: a marvellous landmark experiment

Experiment carried out by Goldhaber et al in 1958

\[ ^{152}\text{Eu} + (\text{K-capture}) \text{e}^- \rightarrow ^{152}\text{Sm}^* + \nu_e \]

\[^{152}\text{Sm}^* \text{ emits } 0.96 \text{ MeV } \gamma \rightarrow \]

1) measurement of direction of \( \gamma \rightarrow \)

measurement of direction of neutrino (back-to-back)

2) measurement of helicity of \( \gamma \) determines helicity of neutrino

3) helicity of \( \gamma \): Compton scattering in iron (below the \(^{152}\text{Eu} \) source) depends on helicity of \( \gamma \) relative to spins of iron; scattering changes \( \gamma \) energy → changing the magnetic field changes spins of iron → changes Compton scattering → measured via resonant absorption in \( \text{Sm}_2\text{O}_3 \)-ring determines helicity of \( \gamma \) → helicity of neutrino \( H(\nu_e) = -1.0 \pm 0.3 \)
Helicity of Neutrino: a marvellous landmark experiment

Count rate in Sm$_2$O$_3$ – analyzer as function of the polarisation of B-field → determines helicity of gamma → helicity of neutrino
Charge Conjugation
(Particle $\rightarrow$ Antiparticle)

• Classical electrodynamics
  - invariant under charge in sign of all electric charges
  - potential, fields reverse sign
  - forces are invariant (charge factor in Lorentz law)

• Elementary particle physics: generalization of ‘changing sign of charge’
  - Charge conjugation $C$ converts particle into antiparticle
    \[ C | p \rangle = | \bar{p} \rangle \]

• More precisely
  - $C$ changes sign of ‘internal’ quantum numbers
    - charge, baryon number, lepton number, strangeness, charm,
    - BUT: mass, energy, momentum, spin NOT affected
  - $C^2 = 1 \Rightarrow$ eigenvalues of $C$ are +1, -1

• Note: most particles are NOT eigenstates of $C$

• Only particles which are their own antiparticles are eigenstates $\rightarrow$ photon, $\pi^0$, $\eta$, $\varphi$, $\ldots$, $\psi$

• $C$ is multiplicative, conserved in electromagnetic and strong processes
Charge Conjugation

- Charge Conjugation is conserved in electromagnetic and strong Inter.
- Mesons: quark-antiquark system
  - one can show: system of (S = ½ particle) • (antiparticle) has
    - eigenstates with \((-1)^{1+S}\)
    - pseudoscalars: \(0, S = 0, C = 1\)
    - vectors: \(0, S = 1, C = -1\)
- Photon: quantum of em field, which changes sign under C: \(C|\gamma\rangle = -1\)
- Examples
  - \(\pi^0 \rightarrow \gamma + \gamma\); \(C\) for \(n\) photons \(C = (-1)^n \Rightarrow \pi^0 \rightarrow 3\gamma\) forbidden; not observed
  - \(p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0 \Rightarrow\) energy distribution for charged pions is on average identical
CP: charge-parity

- Remember the culture shock: weak interactions are not $P$ invariant
  - $\pi^+ \to \mu^+ + \nu_\mu$
  - $\nu_\mu$ emitted is always left-handed $\Rightarrow$ antimuon $\mu^+$ is left-handed
    (pion has $S=0$; muon and neutrino spins opposite)

Weak Interactions are also not invariant under $C$: charge-conjugated reaction

- Charge-conjugated reaction of $C(\pi^+ \to \mu^+ + \nu_\mu) \to \pi^- \to \mu^- + \bar{\nu}_\mu$
  but with helicities unchanged $\Rightarrow$ not possible; $\mu^-$ is not left-handed; always right-handed

- BUT: under combined operation of CP
  - left-handed antimuon $\Rightarrow$ right-handed muon

- Combined operation of CP seems to be the right symmetry operation

- CP Symmetry equivalent to particle-antiparticle symmetry in nature
  - Pauli is happy – ‘die Welt ist wieder in Ordnung’ ……
    for a few years ……
spinning neutrinos and antineutrinos

In weak interactions $P$ and $C$ are “maximally violated” while the combined symmetry under $CP$ is (almost) conserved.

M. Jeitler
• $K^0$ ($d\bar{s}$) and $\bar{K}^0$ ($\bar{d}s$) can be produced in strong interaction processes
  - Kaons are produced in states of unique strangeness
  - $\bar{K}^0$ ($S=-1$) is antiparticle of $K^0$ ($S=+1$)
• Neutral kaons are unstable and decay through weak interaction
  - Experimentally observed: two different decay times!
• Only possible, if these states consist of a superposition of two distinct states with different lifetimes
  - a short-lived one, originally labeled $K_1$
  - a long-lived one, originally labeled $K_2$
• $K^0$ and $\bar{K}^0$ are eigenstates of the Strong Hamiltonian, but not eigenstates of the Weak Hamiltonian
• $K_1$ and $K_2$ are eigenstates of the weak Hamiltonian
Puzzle, Mystery, Beauty of the K-Meson System

- Gell-Mann and Pais (Phys. Rev. 97, 1387, 1955)
- noticed that $K^0$ ($S=+1$) can turn into antiparticle $K^0 (d,s) \leftrightarrow \bar{K}^0 (\bar{d},s)$, through second order weak interaction

Feynman diagrams in modern formulation

- Particles, normally observed in laboratory, are linear combinations of these two states
Neutral K-System

- K’s are pseudoscalars

\[ P|K^0\rangle = -|K^0\rangle \quad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle \]
\[ C|K^0\rangle = |\bar{K}^0\rangle \quad C|\bar{K}^0\rangle = |K^0\rangle \]
\[ CP|K^0\rangle = -|\bar{K}^0\rangle \quad CP|\bar{K}^0\rangle = -|K^0\rangle \]

- The normalized eigenstates of CP are

\[ |K_1\rangle = \left(\frac{1}{\sqrt{2}}\right)(|K^0\rangle - |\bar{K}^0\rangle) \quad |K_2\rangle = \left(\frac{1}{\sqrt{2}}\right)(|K^0\rangle + |\bar{K}^0\rangle) \]
\[ CP|K_1\rangle = |K_1\rangle \quad CP|K_2\rangle = -|K_2\rangle \]

- If CP is conserved in weak interactions
  - \( K_1 \to \) can decay only in \( CP = +1 \) state
  - \( K_2 \to \) can decay only in \( CP = -1 \) state

- Kaons typically decay into \( \pi^0 \) state

  2 \( \pi \) state (CP = +1)
  3 \( \pi \) state (CP = -1)

- Conclusion: \( K_1 \to 2\pi \), \( K_2 \to 3\pi \)
$K_1, K_2$

- $2\pi$ – decay is much faster (more energy, i.e. more phase space available)

- Start with $K^0$-beam

\[
|K^0\rangle = \left(\frac{1}{\sqrt{2}}\right)(|K_1\rangle - |K_2\rangle)
\]

- $|K_1\rangle$ component will decay quickly, leaving more $|K_2\rangle$‘s

- In Cronin’s memoirs

So these gentlemen, Gell-Mann and Pais, predicted that in addition to the short-lived K mesons, there should be long-lived K mesons. They did it beautifully, elegantly and simply. I think theirs is a paper one should read sometimes just for its pure beauty of reasoning. It was published in the Physical Review* in 1955. A very lovely thing! You get shivers up and down your spine, especially when you find you understand it. At the time, many of the most distinguished theoreticians thought this prediction was really

*baloney* (‘*Unsinn*’).

... it was not baloney

- 1955: Lederman and collaborators discover $K_2$ meson
  \[
  \tau_1 = 0.895 \times 10^{-10} \text{ sec} \\
  \tau_2 = 5.11 \times 10^{-8} \text{ sec}
  \]

- Note: $K_1$ and $K_2$ are NOT antiparticles of one another
  
  ($K_0$ and $\bar{K}_0$ are antiparticles of one another)

  $K_1$ is its own antiparticle $C = -1$
  $K_2$ is its own antiparticle $C = +1$

- They differ by a tiny mass difference
  \[
  m_2 - m_1 = 3.48 \times 10^{-6} \text{ eV} \quad (\sim 10^{-11} \text{ of electron mass})
  \]
What is a ‘Particle’?

- Kaons are produced by strong interactions, in eigenstates of strong Hamiltonian, in eigenstates of strangeness \((K^0 \text{ and } \bar{K}^0)\).

- Kaons decay by the weak interaction, as eigenstates of CP \((K_1, K_2)\).

- What is the real particle? Characterized by unique life time?

- Analogy with polarized light
  - linear polarization can be regarded as superposition of left-circular and right-circular polarization
  - traversal of medium, which preferentially absorbs right-circularly polarized light \(\Rightarrow\) linearly polarized light will become left-polarized \(K^0\) beams \(\Rightarrow\) \(K_2\) beams
The Cultural Revolution: CP violation

• 1964: Cronin, Fitch and collaborators observe CP violation

• $K_0$ - beam: by letting the $K_1$ component decay $\Rightarrow$ can produce arbitrarily pure $K_2$ - beam; $K_2$ is a CP=-1 state; can only decay into CP=-1 (3 pions), if CP is conserved

• Observation:
  - observed: 22700 $3\pi$-decay
  - observed: 45 $2\pi$-decay

• Long-lived component is NOT perfect eigenstate of CP, contains a small admixture of $K_1$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2\rangle + \varepsilon |K_1\rangle)$$
$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1\rangle + \varepsilon |K_2\rangle)$$

• measure of departure from perfect CP invariance is $\varepsilon$: $\varepsilon = 2.24 \times 10^{-3}$
Cronin et al experiment

- spark chambers
- scintillators
- Cherenkov detectors

the first signal:

\[ K_L \rightarrow \pi^+ \pi^- \]
• Parity violation is treated in W.I. easily because it is maximally violated
• CP violation in contrast is a very small effect
• In the ‘Standard Model’ it was incorporated in the ‘Cabbibo-Kobayashi-Maskawa’ (CKM) mixing matrix ⇒
• 1973: Kobayashi, Maskawa: show how it could be incorporated, but requiring THREE generations of quarks ! (Nobel Prize in 2008)
• Even more dramatic: 41% of $K_L$ decay semileptonically
  - $K_L \rightarrow \pi^+ + e^- + \bar{\nu}_{e}$ and (CP) $\pi^- + e^+ + \nu_{e}$
  - if CP is good symmetry: the two decay rates are equal
  - experimentally: $K_L \rightarrow \pi^- + e^+ + \nu_{e}$ preferred by 1 in $3.3 \times 10^{-3}$
• Absolute distinction between Matter and Antimatter
  ‘Matter’: defined by charge of lepton produced preferentially in the decay of $K_L$!
Beauty Factories

- CP violation occurs also in neutral $B$-meson system

- ‘$B$-factories’: $e^+e^-$ colliders, optimized for $B\bar{B}$ - production
  - constructed at SLAC (BaBar Experiment)
  - constructed at KEK (BELLE Experiment)

- The precision experiments confirmed the CKM Theory $\Rightarrow$ cited in the Nobel Prize Award

- HEPHY is a major partner in BELLE and

- Leading partner in BELLE II (aim for much higher sensitivity)
  - one research area with challenging opportunities for project diploma, dissertation work
    - talk to C. Schwanda (BELLE II Project Leader) or C. Fabjan
Time Reversal and CTP Theorem

- CP is violated: what about T invariance?

- T invariance very difficult to test experimentally
  - expected to be violated in W.I. ⇒ usually signal overwhelmed by em and strong interaction

- Classic example:
  - electric dipole moment $d$ of elementary particle (neutron)
  - $d$ points along spin $S$
  - $d$ is vector, $S$ is pseudovector
  - $d \neq 0$ ⇒ violation of P
  - $S$ changes sign under T, $d$ does not
  - $d \neq 0$ ⇒ violation of T
Neutron Electric Dipole Moment (nEDM)

Assume neutron is globally neutral, but has positive and negative charge distribution resulting in electric dipole moment. Time reversal changes spin direction, but does not change charge distribution. Parity changes EDM, but not spin. $n_{EDM} \neq 0 \rightarrow$ System not symmetric with respect to initial system. → Time and Parity invariance is violated. → Given CPT invariance, also CP is violated. Present limit $n_{EDM} < 3 \times 10^{-26}$ e.cm. Standard Model (due to CP violation) $n_{EDM} \approx 10^{-32}$ e.cm. Extension of SM: $n_{EDM} \leq 10^{-28}$ e.cm. Very active field of experimental research.
Measure $T$-violation by comparing $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$

Compare rates for neutral kaons which are created as $K^0$ and decay as $\bar{K}^0$ with the inverse process:
Direct measurement of T-violation by CPLEAR at CERN

\[
A_T = \frac{R(\bar{K}^0 \rightarrow K^0) - R(K^0 \rightarrow \bar{K}^0)}{R(\bar{K}^0 \rightarrow K^0) + R(K^0 \rightarrow \bar{K}^0)} = 4\Re \varepsilon_T
\]

\[A_T = (6.6 \pm 1.3_{\text{stat.}} \pm 1.0_{\text{syst.}}) \times 10^{-3}\]

"First direct" measurement of time reversal non-invariance!


M. Jeitler