Extracting HQET parameters from moments

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[1] Falk and Luke, PRD 57:424,1998
[2] Uraltsev et al.
[3] Bauer et al., PRD67:054012,2003

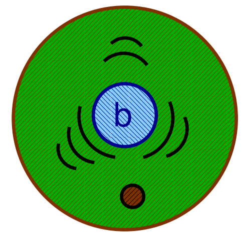
$\langle m_X^2 \rangle$: Inclusive Semileptonic B Decays

- Heavy Quark Expansion gives (schematically) decay rate:

$$\Gamma(b \to c\ell\bar{\nu}) \propto \frac{G_F^2 |V_{cb}|^2}{192\pi^3} m_b^5 \left[1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \frac{\alpha_S}{\pi}(\ldots) + \ldots \quad \right]$$

- Non-perturbative parameters
  - ▷ $\overline{\Lambda}$: interactions of light degrees of freedom with

    b-quark : $m_B = m_b + \overline{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots$



  - ▷ λ_1 : $(-)$ kinetic energy squared of b-quark in B-meson
  - ▷ λ_2 : chromo-magnetic coupling of b-quark spin to "brown muck"
  - ▷ $\ldots$ more parameters at higher order, (completely) unknown
- Similar expansions for other observables
  - ▷ moments of hadronic mass (-squared) distributions: $\langle m_X^2 \rangle|_{p^* > p^*_{min}}$
  - ▷ moments of lepton energy distributions, $\ldots$
  - ▷ with kinematic cuts, various schemes ($\overline{MS}$ [1], kinetic [2], $1S$ [3], $\ldots$)
- Combined fits to moments and $\Gamma_{sl} \to |V_{cb}|, \overline{\Lambda}, m_b, \ldots$

Determining $|V_{cb}|$ from Inclusive Semileptonic B Decays

Using the Heavy Quark Expansion to determine $|V_{cb}|$: problem:

- Inclusive observables are written in terms of a double expansion in α_S and $1/M_B$ (M_B is the observable B meson mass)
  - Nonperturbative QCD parameters enter at each order in the expansion
  - Current goal is to determine these parameters from experimental measurements
- The b quark mass is related to M_B and three nonperturbative QCD parameters:

$$M_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots$$

$$M_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} + \ldots$$

- Intuitively:
  - $\bar{\Lambda}$ is the energy of the light quark and gluon degrees of freedom
  - $-\lambda_1$ is the average of the square of the b quark momentum
  - λ_2/m_b is the hyperfine interaction of the b quark and light degrees of freedom
- Determine λ_2 from $M_{B^*} - M_B \approx 46$ MeV$/c^2$
- Determine $\bar{\Lambda}$ and λ_1 from hadronic mass moments in $\bar{B} \to X_c \ell \bar{\nu}$ decay and photon energy moments in $B \to X_s \gamma$ decay

Determining $|V_{cb}|$ from Hadronic Mass Moments and $B \to X_s\gamma$

$\Gamma(\bar{B} \to X_c \ell \bar{\nu})$ can be written in the form:

$$\Gamma^c_{SL} = \frac{G_F^2 \, |V_{cb}|^2 \, M_B^5}{192\pi^3}\Big[\mathcal{G}_0 + \frac{1}{M_B}\mathcal{G}_1(\bar{\Lambda}) + \frac{1}{M_B^2}\mathcal{G}_2(\bar{\Lambda}, \lambda_1, \lambda_2) + \frac{1}{M_B^3}\mathcal{G}_3(\bar{\Lambda}, \lambda_1, \lambda_2 | \rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4) + \mathcal{O}\left(\frac{1}{M_B^4}\right)\Big]$$

- $\bar{\Lambda}, \lambda_1, \lambda_2, \rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$ are all nonperturbative parameters,
- $\mathcal{G}_n$ are polynomials of order $\le n$ in $\bar{\Lambda}, \lambda_1, \lambda_2$, and power series in α_S
- $\mathcal{G}_3$ is linear in $\rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$, and we use bounds from theoretical estimates.
- We determine $\bar{\Lambda}$ and λ_1 from $\bar{B} \to X_c \ell \bar{\nu}$ and $B \to X_s\gamma$ decays

There are similar polynomial expressions for the moments

- $\langle (M_X^2 - \bar{M}_D^2) \rangle$ of the $\bar{B} \to X_c \ell \bar{\nu}$ hadronic mass (M_X) spectrum with coefficients $\mathcal{M}_n(\bar{\Lambda}, \lambda_1, \lambda_2, \ldots)$
  - M_X is the mass of the hadronic system in $\bar{B} \to X_c \ell \bar{\nu}$ decay
  - $\bar{M}_D \equiv (M_D + 3M_{D^*})/4)$ – spin averaged $D^{(*)}$ mass
- $\langle E_\gamma \rangle$ of the $b \to s\gamma$ photon energy (E_γ) spectrum with coefficients $\mathcal{E}_n(\bar{\Lambda}, \lambda_1, \lambda_2, \ldots)$.

Different measurements in literature

Hadronic mass moments – Moments of $m(X)$ in $B \to Xl\nu$

- CLEO 2001 [PRL 87, 251808 (2001)]
  - inclusive neutrino reconstruction, $m(X)^2 = (p_B - p_l - p_\nu)^2$
  - hard lepton momentum cut, $p_l > 1.5$ GeV$/c$
- BABAR 2003 (summer conference paper)
  - full reconstruction of the other B, $p_l > 0.9$ GeV$/c$

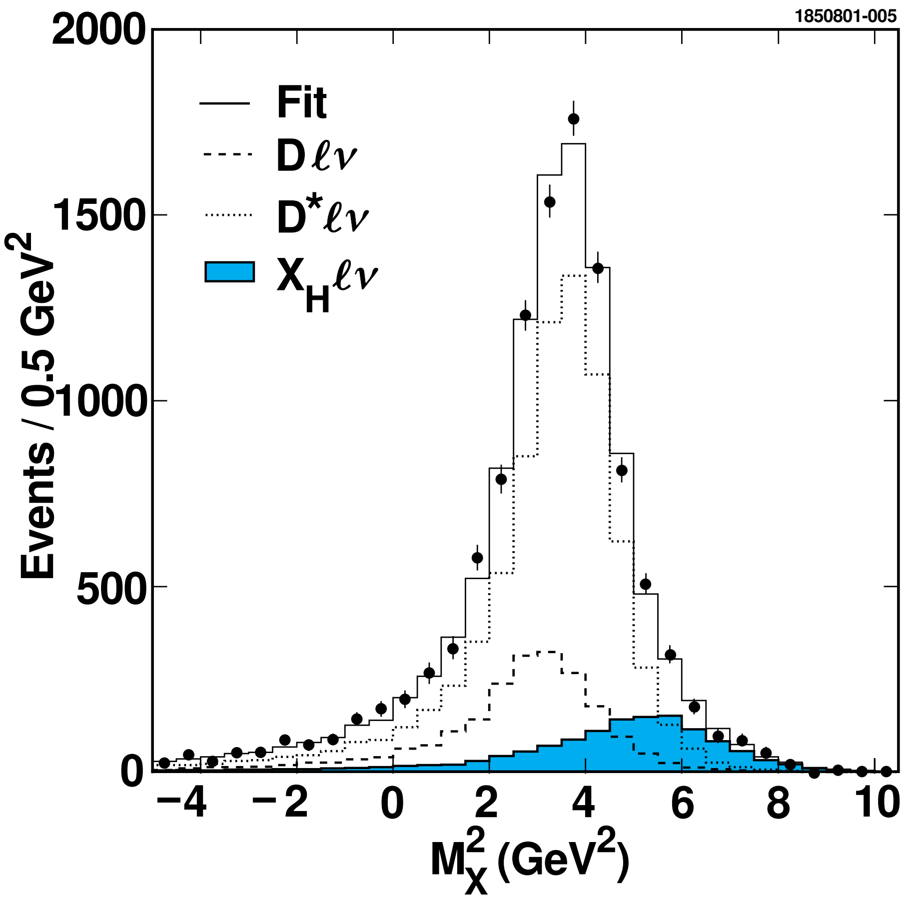
Lepton moment moments – Moments of $p(l)$ in $B \to Xl\nu$

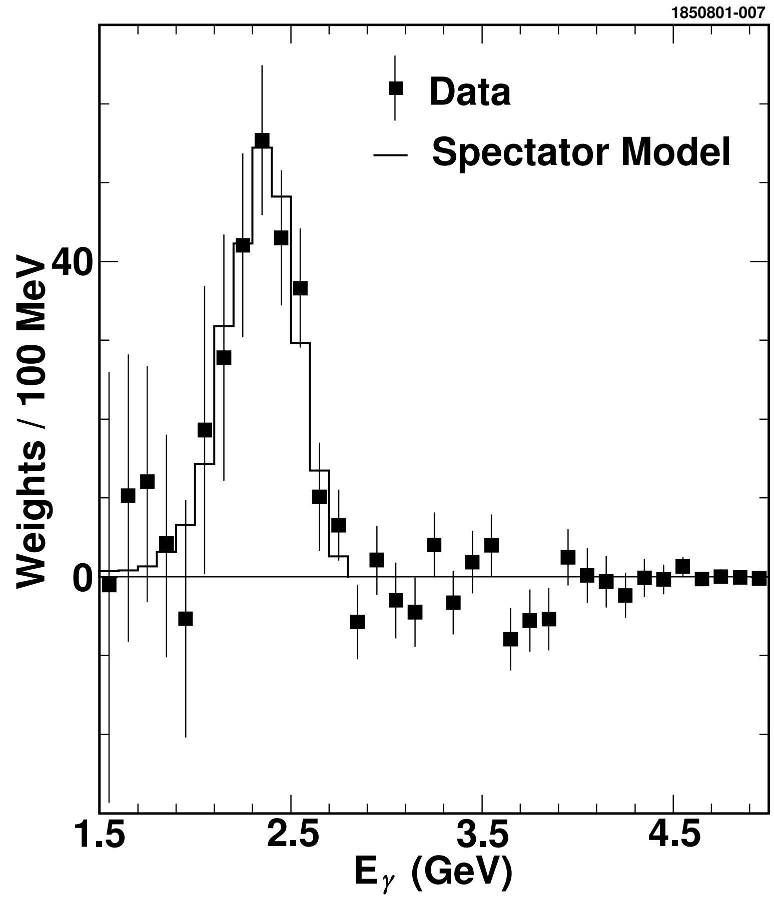
- CLEO 2003 [PRD 67, 072001 (2003)]

Both hadronic and leptonic moments

- DELPHI (summer 2002 and 2003 conference paper)

Determining $|V_{cb}|$ from Hadronic Mass Moments and $B \to X_s\gamma$





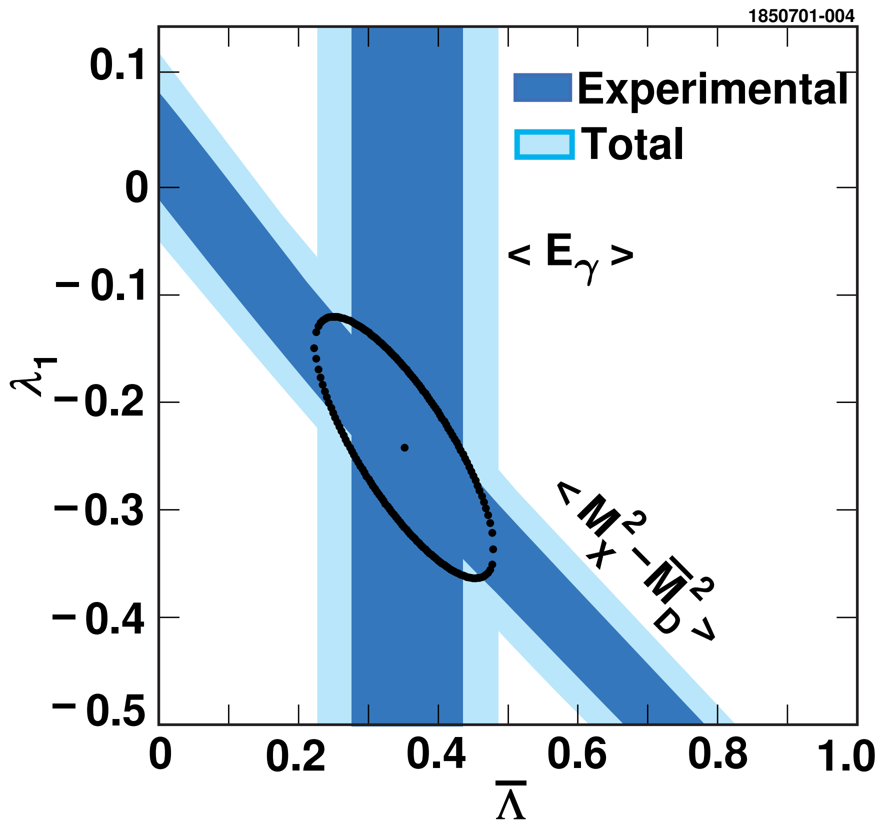
$$\langle (M_X^2 - \bar{M}_D^2) \rangle = 0.251 \pm 0.023 \pm 0.062 \text{ GeV}^2$$
$$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 0.639 \pm 0.056 \pm 0.178 \text{ GeV}^4$$

$$\langle E_\gamma \rangle = 2.346 \pm 0.032 \pm 0.011 \text{ GeV}$$
$$\langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle = 0.0226 \pm 0.0066 \pm 0.0020 \text{ GeV}^2$$

Determining $|V_{cb}|$ from Hadronic Mass Moments and $B \to X_s\gamma$

The intersection of the E_γ and M_X moments yields $\bar{\Lambda}$ and λ_1.

$$\begin{aligned}\bar{\Lambda} &= 0.35 \pm 0.07 \pm 0.10 \text{ GeV} \\ \lambda_1 &= -0.238 \pm 0.071 \pm 0.078 \text{ GeV}^2 \\ & \qquad\qquad\quad (M) \qquad (T)\end{aligned}$$



Then the expression for Γ^c_{SL} yields

$$|V_{cb}| = (40.4 \pm 0.9 \pm 0.5 \pm 0.8) \times 10^{-3}$$

(M) (Γ) (T)

Errors are due to

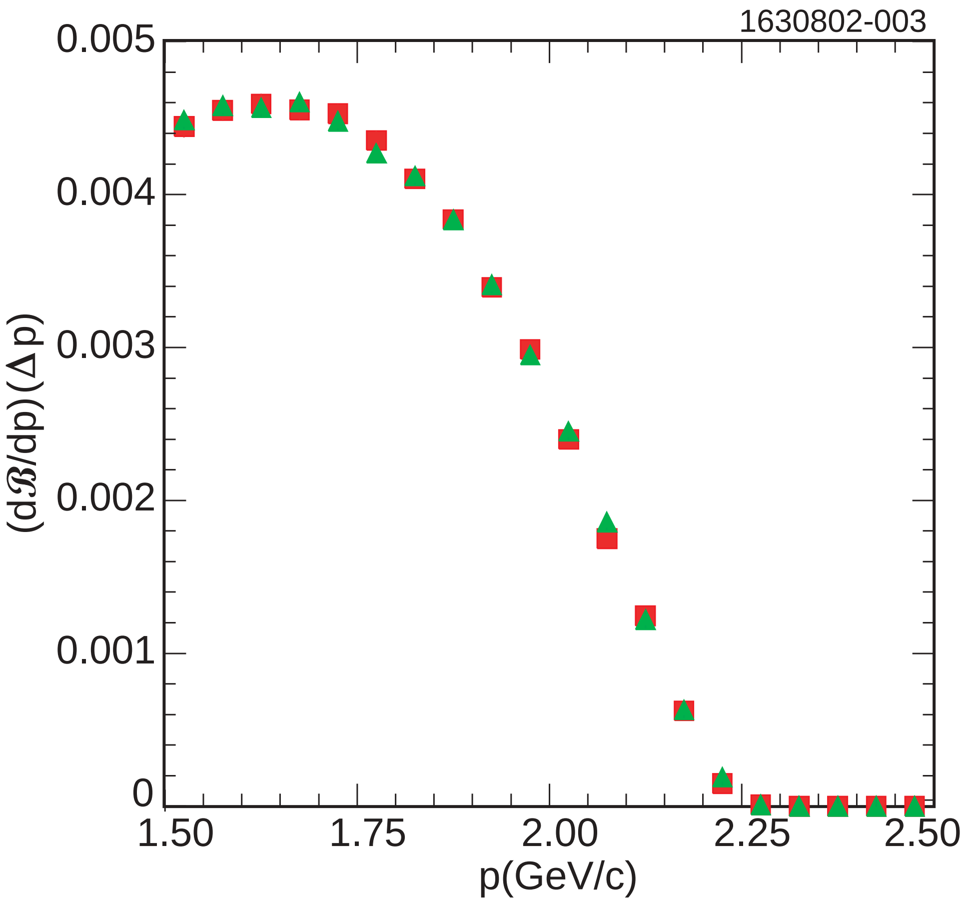
(M) moment uncertainties,

(Γ) Γ^c_{SL} uncertainties, and

(T) α_s scale and ignoring the $\mathcal{O}(1/M_B^3)$ term which contains the estimated parameters

Even with this measurement of QCD parameters, the residual theoretical uncertainties (T) are comparable to the experimental errors (M) and (Γ).

Measuring Moments of the Inclusive $\bar{B} \to X\ell\bar{\nu}$ Spectra



Measured Moments

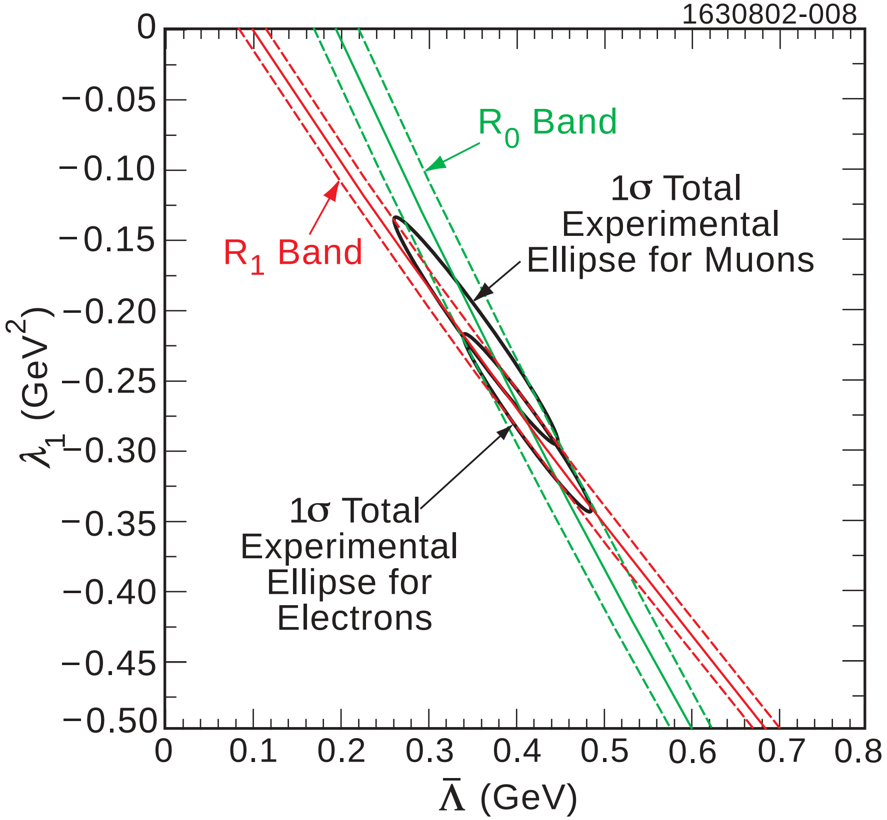
R_0	e	$0.6184 \pm 0.0016 \pm 0.0017$	
R_0	μ	$0.6189 \pm 0.0023 \pm 0.0020$	
R_0	ℓ	$0.6187 \pm 0.0014 \pm 0.0016$	
R_1	e	$1.7817 \pm 0.0008 \pm 0.0010$	GeV
R_1	μ	$1.7802 \pm 0.0011 \pm 0.0011$	GeV
R_1	ℓ	$1.7810 \pm 0.0007 \pm 0.0009$	

Dominant systematic errors:

- $b \to c \to s(d)\ell\nu$ contribution
- Lepton identification
- Electroweak radiative corrections
- Absolute momentum scale uncertainty

Determining $\bar{\Lambda}$ and λ_1 from Moments of the Lepton Spectra

$R_0(e)$ and $R_1(e)$ in the $\lambda_1 - \bar{\Lambda}$ plane



Error bands are $\pm 1\sigma$ total experimental

$\bar{\Lambda}$ and λ_1 from R_0 and R_1

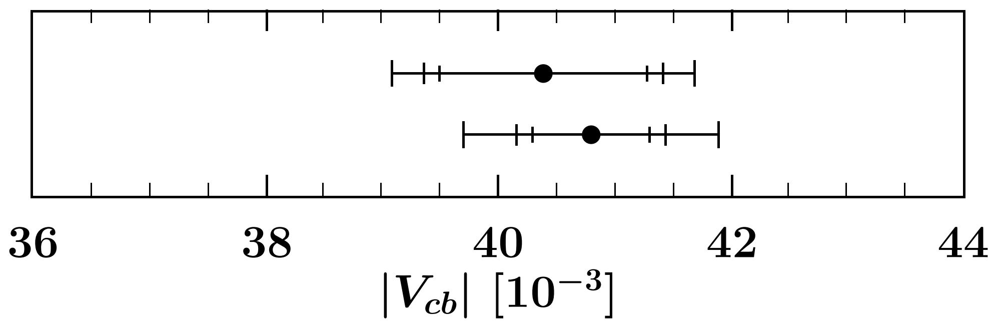
$\bar{\Lambda}$	e	$+0.41 \pm 0.04 \pm 0.06 \pm 0.12$	GeV
$\bar{\Lambda}$	μ	$+0.36 \pm 0.06 \pm 0.08 \pm 0.12$	GeV
$\bar{\Lambda}$	ℓ	$+0.39 \pm 0.03 \pm 0.06 \pm 0.12$	GeV
λ_1	e	$-0.28 \pm 0.03 \pm 0.06 \pm 0.14$	GeV^2
λ_1	μ	$-0.22 \pm 0.04 \pm 0.07 \pm 0.14$	GeV^2
λ_1	ℓ	$-0.25 \pm 0.02 \pm 0.05 \pm 0.14$	GeV^2

- Errors are stat, sys, and theory
  - $1/\bar{M}_B^3$ uncertainty dominates the theoretical error

Comparing $|V_{cb}|$ from Different Moments

Inclusive Measurements of $|V_{cb}|$

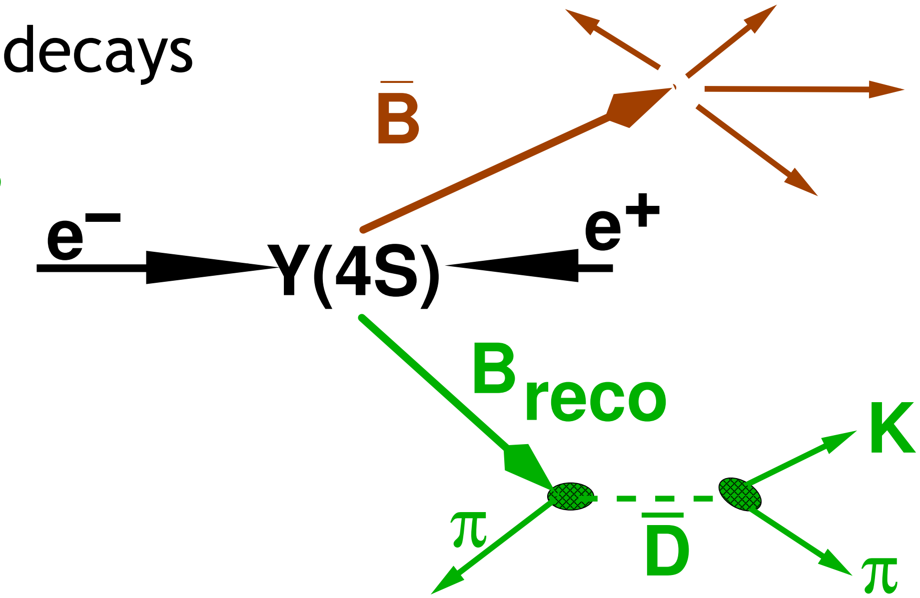
Moments Used	$|V_{cb}|$ $[10^{-3}]$
M_X^2 and E_γ	$40.4 \pm 0.9 \pm 0.5 \pm 0.8$
E_ℓ	$40.8 \pm 0.5 \pm 0.4 \pm 0.9$



$\langle m_X^2 \rangle$: 'Recoil' Physics

- Fully reconstructed hadronic B-decays

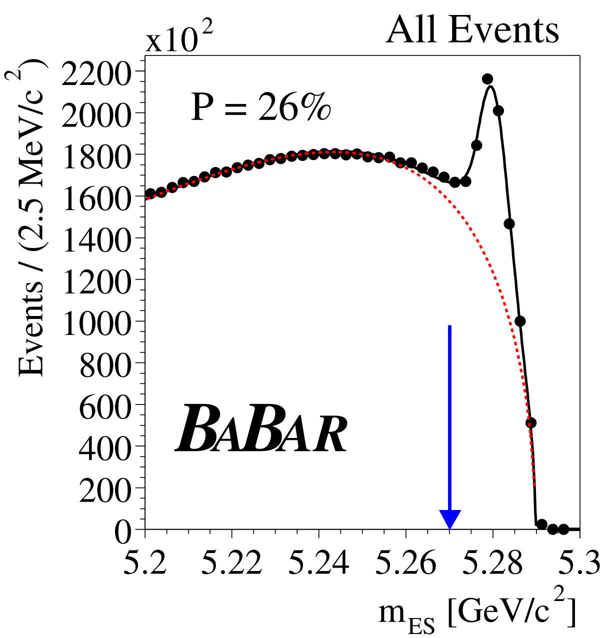
$$B \rightarrow D^{(*)}(n_1\pi^{\pm} + n_2K^{\pm} + n_3K_S^0 + n_4\pi^0),$$
$$n_1 + n_2 \leq 5, n_3 \leq 2, n_4 \leq 2$$

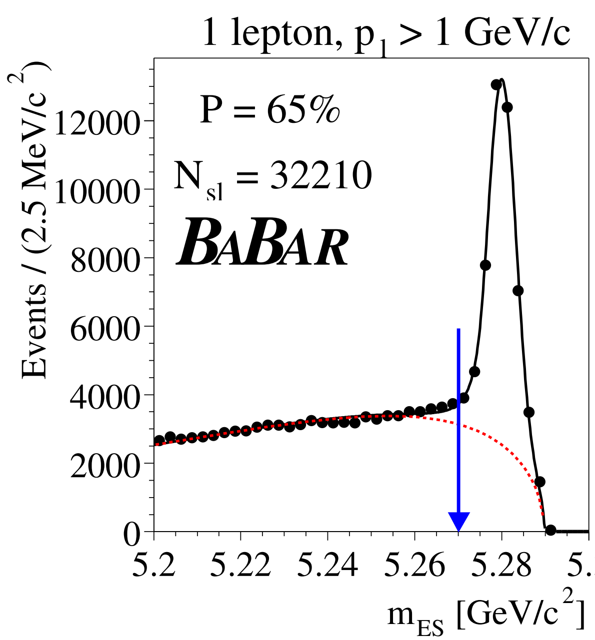


- Advantage at $\Upsilon(4S)$:

$$m_{ES} = \sqrt{E_{beam}^2 - \vec{p}_B^{\,2}}$$
$$\Delta E = E_B - E_{beam}$$

- Yield $\sim 4000B/\text{fb}^{-1}$
  - ▷ $N_{B^0} : N_{B^+} \sim 1 : 2$





- Benefits:
  - ▷ momentum of B
  - ▷ flavor of B
  - ▷ background reduction
  - ▷ m_{ES} sideband

$\langle m_X^2 \rangle$: Event Selection

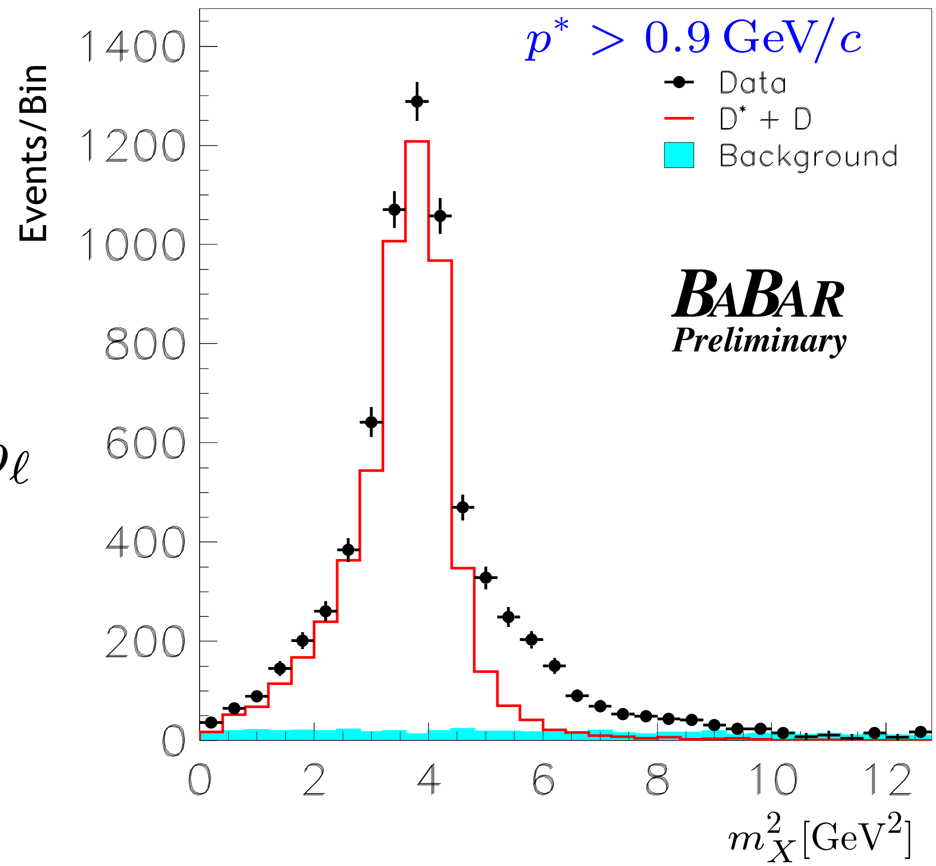
- Goal: Measure m_X^2 distribution and its first moment for different lepton momentum cuts p^*_{min}
  $\rightarrow$ bands with varying slopes in $(\lambda_1, \overline{\Lambda})$ plane

- Tag events with B_{reco} and
  - ▷ Lepton ℓ with $p^* > 0.9\,\mathrm{GeV}/c$
  - ▷ $Q_\ell \cdot Q_{b(B_{reco})} < 0$
  - ▷ $|Q_{tot}| = |Q_{B_{reco}} + Q_\ell + Q_X| \leq 1$

  $\rightarrow$ 7100 signal events

- Neutrino reconstruction via
  $p_{miss} = p_{\Upsilon(4S)} - p_{B_{reco}} - p_X - p_\ell$
  - ▷ $E_{miss} > 0.5\,\mathrm{GeV}$
  - ▷ $|\vec{p}_{miss}| > 0.5\,\mathrm{GeV}$
  - ▷ $|E_{miss} - |\vec{p}_{miss}|| < 0.5\,\mathrm{GeV}$

- 2C kinematic fit
  $\rightarrow \sigma(m_X) \sim 350\,\mathrm{MeV}/c^2$

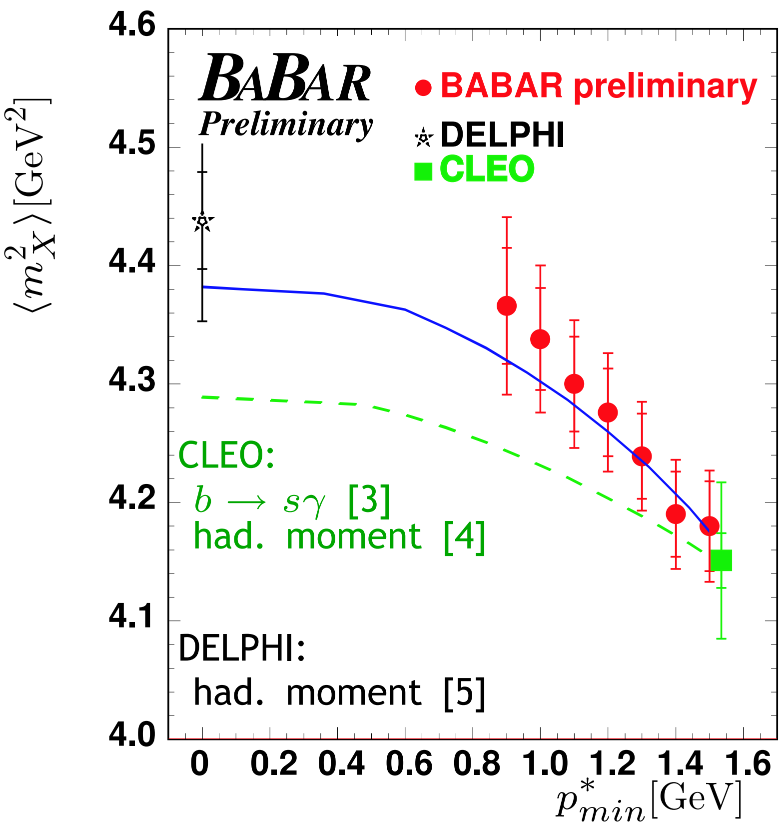


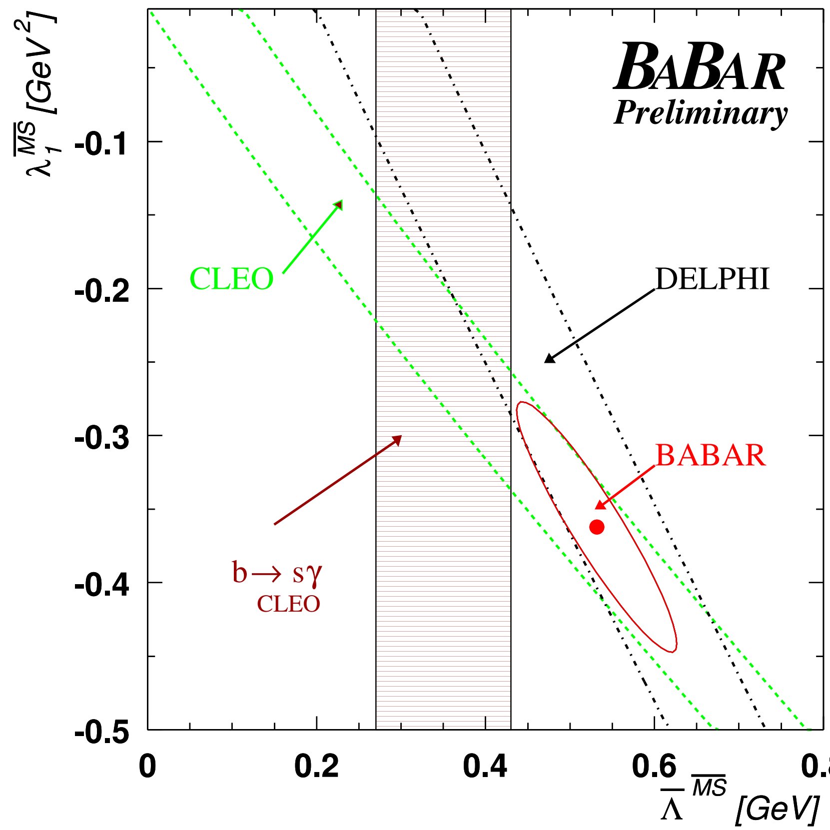
[1] Falk and Luke, PRD 57:424,1998
[2] BABAR, EPS285
[3] CLEO, PRL87:251807, 2001
[4] CLEO, PRL87:251808, 2001
[5] DELPHI, hep-ex/0210046

$\langle m_X^2 \rangle$: Fit to BABAR Moments

- Fit [1] to all BABAR moments [2]:
  - take into account correlations
  - fix $\lambda_2 = 0.128\,\mathrm{GeV}^2$, $\mathcal{T}_i = 0$, $\rho_1 = \frac{1}{2}(0.5)^3\,\mathrm{GeV}^3$, $\rho_2 = \lambda_2$
  - illustrations without $1/m_B^3$ errors

$$\overline{\Lambda}^{\overline{MS}} = 0.53 \pm 0.09_{exp}\,\mathrm{GeV}$$
$$\lambda_1^{\overline{MS}} = -0.36 \pm 0.09_{exp}\,\mathrm{GeV}^2$$





- All hadronic moments consistent

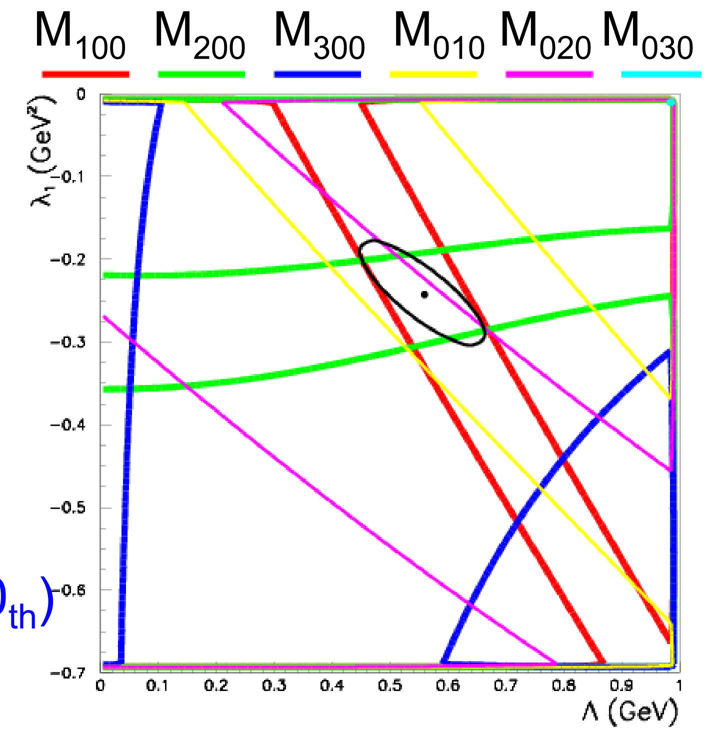
Moment measurements: CLEO, DELPHI, BABAR

DELPHI 2003 determines M_{i00} by reconstructing $B \rightarrow D^{**} l\nu$ events from 34 M Z at LEP. $\langle m_X^2 - \overline{m}_D^2 \rangle = (0.647 \pm 0.046 \pm 0.093)\ GeV^2$, $\overline{m}_D = (m_D + 3 m_{D^*})/4$. Also results for M_{200}, M_{300}

2002 they obtained $\langle E_l \rangle = (1.383 \pm 0.012 \pm 0.009)$ GeV, also M_{020}, M_{030}.

They fit in two mass schemes, with $\overline{MS}$ scheme:

Fit Parameter	Fit Values	Fit Uncertainty	Syst. moments	Syst. theory
$\bar{\Lambda}$ (GeV)	0.542	± 0.065	± 0.087	± 0.04
λ_1 (GeV^2)	-0.238	± 0.055	± 0.028	± 0.06

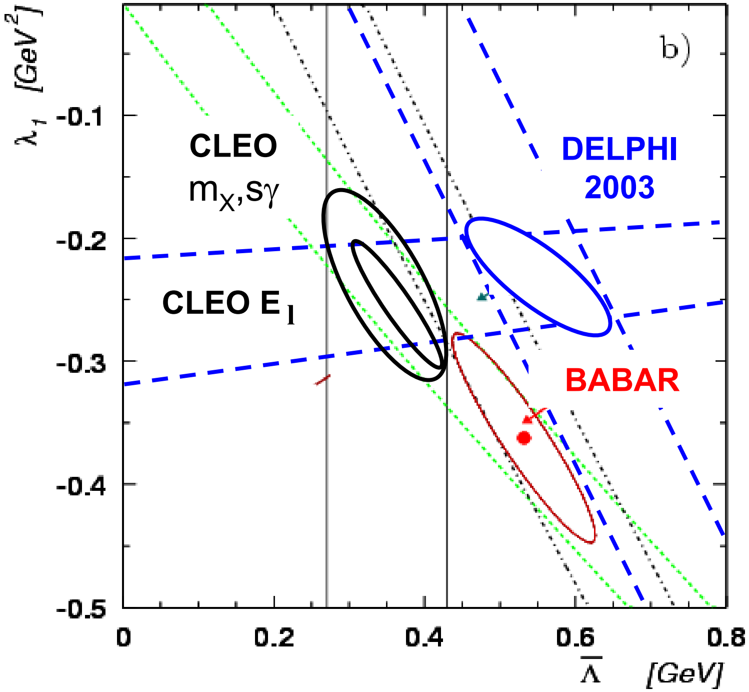


With kinetic scheme:

$|V_{cb}| = 0.0429\ (1 \pm 0.012_{\Gamma sl} \pm 0.019_{fit} \pm 0.010_{th})$

$\boldsymbol{BF}(B \rightarrow l\nu X)$ readjusted to 10.9%

Summary V_{cb} inclusive:



Moment measurements

of $\frac{d^3\Gamma(B \to l\nu\ X_c)}{dm_X^2\ dE_l\ dq^2}$

and HQET/OPE offer the potential to determine $|V_{cb}|$ with $\sigma < 2\%$.

Duties for experiment:

Identify inconsistency areas.

for theory: Identify best scheme (1S ?), get terms with α_s/m_b^2 and more $1/m_b^3$ and α_s^2 terms ...

$|V_{cb}|_{incl} = 0.0429\ (1 \pm 0.012_{\Gamma sl} \pm 0.019_{fit} \pm 0.010_{th})$ DELPHI, kin

$0.0414\ (1 \pm 0.012_{\Gamma sl} \pm 0.022_{fit} \pm 0.020_{th})$ CLEO m, pole mass

$0.0418\ (1 \pm 0.012_{\Gamma sl} \pm 0.012_{fit} \pm 0.022_{th})$ CLEO E, pole mass

$0.0421\ (1 \pm 0.025_{exp} \pm 0.017_{th})$ BABAR, 1S

My average: $|V_{cb}|_{incl} = 0.0421 \pm 0.0013\ \ (3.0\%)$