How to test entanglement for mesons?

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Bell inequalities

Photons (Experiments: Aspect[1], Weihs[2])

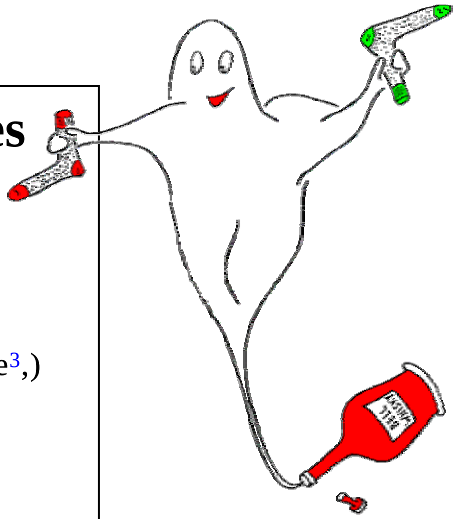
Atoms (Experiment: Rowe[3],)

Neutrons (Experiment: Hasegawa[4])

K-mesons (??)

B-mesons (??)

....



Decoherence model

Photons (realisation with Zeilinger group)

Atoms (???)

Neutrons (realisation with Rauch group)

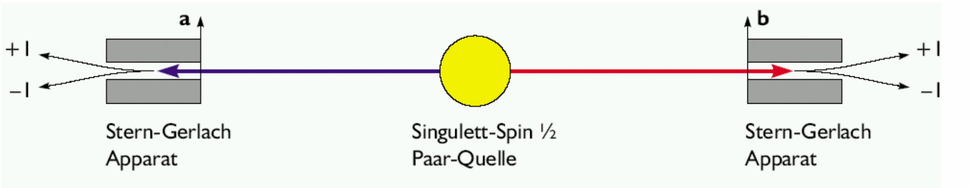
K-mesons (realisation with DAPHNE group)

B-mesons (realisation with HEPHY group ?)

Bertlmann, Grimus and Hiesmayr, Phys. Rev. D, 60, 114032 (1999)
Bertlmann, Durstberger and Hiesmayr, Phys. Rev. A 68 (2003) 012111

1. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
2. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998).
3. M. Rowe et al., Nature 409, 791 (2001).
4. Y. Hasegawa, R. Loidl, G. Bardurek, M. Baron and H. Rauch, Nature 425, 45 (2003).

EPR-Problem



$$\left|\psi^{-}\right\rangle = \tfrac{1}{\sqrt{2}}\left\{\left|\Uparrow\right\rangle_{l} \otimes \left|\Downarrow\right\rangle_{r} - \left|\Downarrow\right\rangle_{l} \otimes \left|\Uparrow\right\rangle_{r}\right\}$$

Einstein:

Es darf keine spukhafte Fernwirkung existieren!!
(There exists no instantaneously influence between two in space separated particles.)

What are Bell inequalities?


John S. Bell

All local realistic theories (LRT) are constrained by certain inequalities.

Assumptions:

REALISM
LOCALITY
INDUCTION

⇩

Local realistic theories (LRT)

BELL INEQUALITIES always fulfilled!

Quantum theory

BELL INEQUALITIES not fulfilled (in general)!

Test via experiments

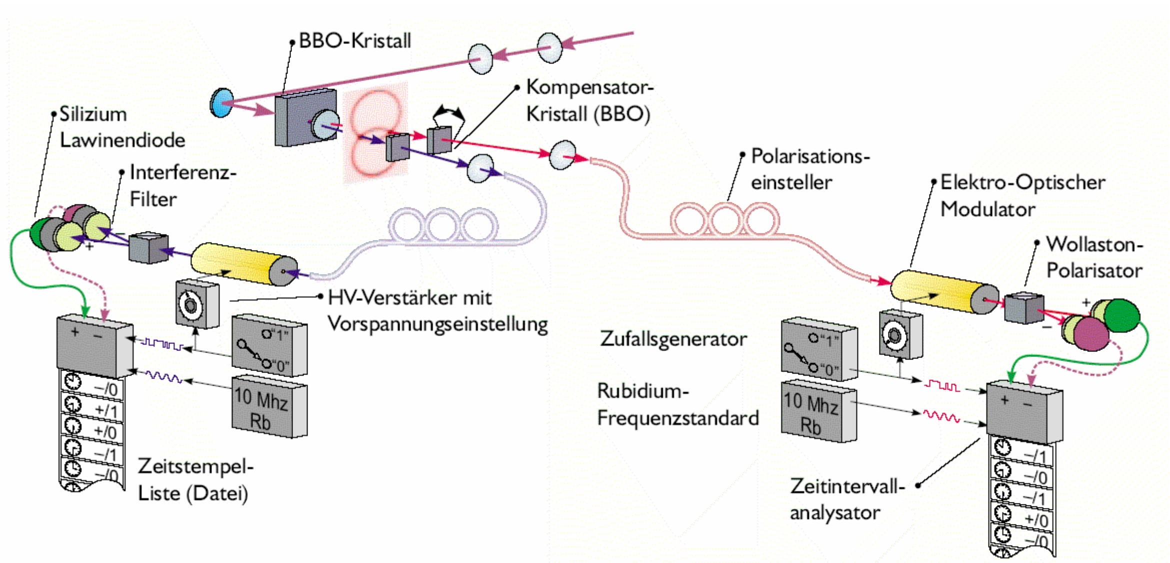
⇩

Comparison with theory

⇩

Correction of the assumption

CHSH-inequality for photons



$$|\psi^-\rangle \cong \{ |H\rangle_l \otimes |V\rangle_r - |V\rangle_l \otimes |H\rangle_r \}$$

CHSH-inequality:

$$S(\alpha,\beta,\gamma,\delta) = |E(\alpha;\beta) - E(\alpha;\gamma)| + |E(\delta;\beta) + E(\delta;\gamma)| \leq 2$$

Quantum theory:

$$E^{QM}(\alpha,\beta) = -\cos(2(\beta - \alpha))$$

Maximal value of S for QT:

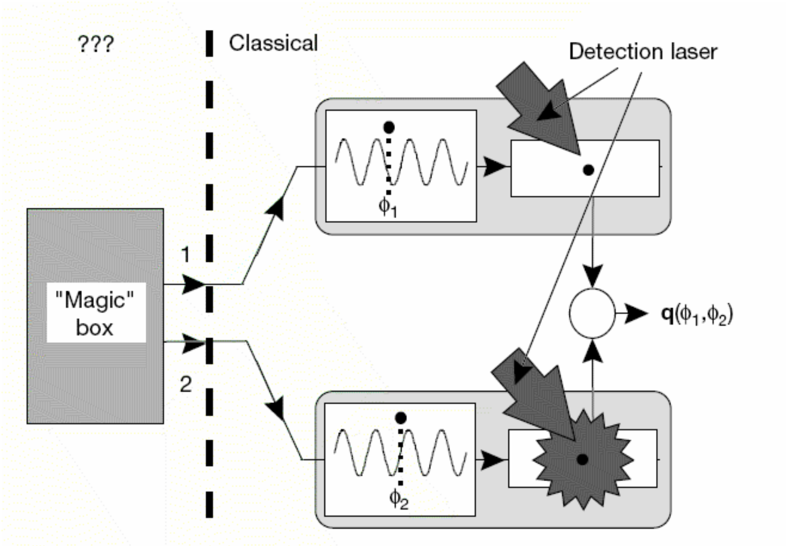
$$\max S = 2\sqrt{2}$$

LOOPHOLES:

- **Locality** (~no "communication" over macroscopic distances possible) **!!CLOSED!**
- **detection efficiency** (~fair sampling hypothesis)

!!NOT CLOSED!!

CHSH-inequality for atoms

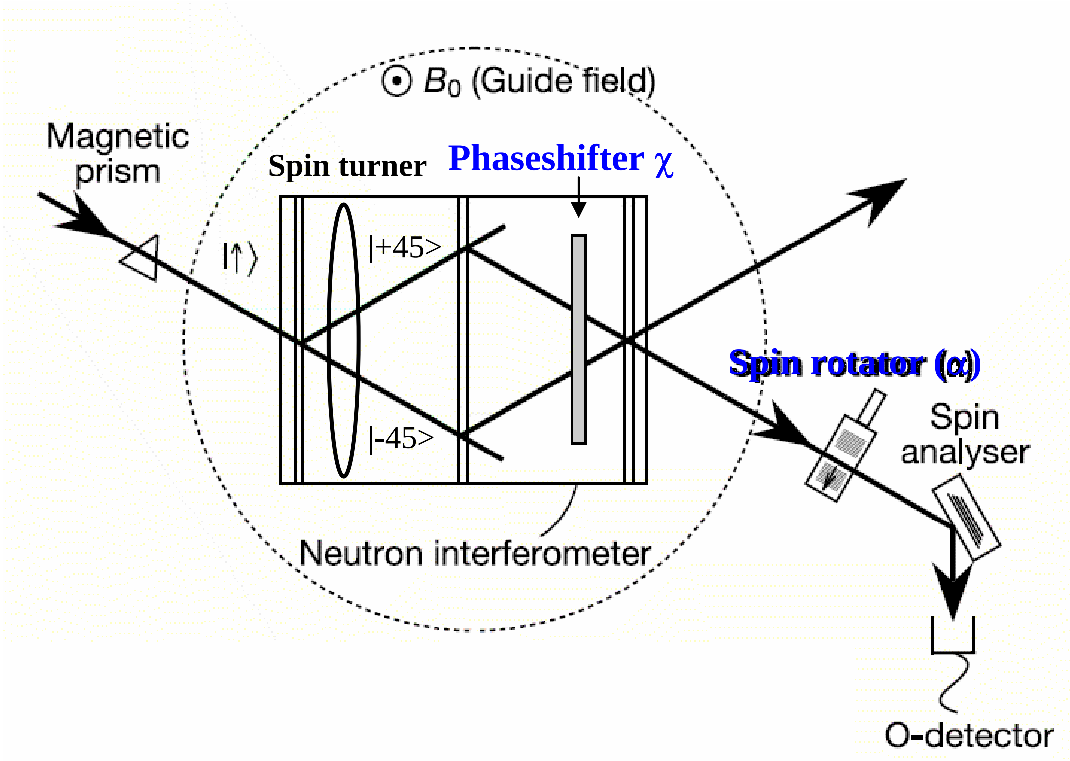


$$S_{\exp} = 2.25 \pm 0.03$$

LOOPHOLES:

- **Locality** (~no "communication" over macroscopic distances possible) **!!NOT CLOSED!!**
- **detection efficiency** (~fair sampling hypothesis)! **!! CLOSED!!**

CHSH-inequality for single neutrons



$$\left|\psi^-\right\rangle \cong \left\{\left|\boldsymbol{I}\right\rangle \otimes \left|\Uparrow\right\rangle - \left|\boldsymbol{II}\right\rangle \otimes \left|\Downarrow\right\rangle\right\}$$

$$S_{\exp} = 2.051 \pm 0.019$$

LOOPHOLES:

- **NONCONTEXTUALITY** (~the value of an observable (spin) does NOT depend on the co-measured observable (path)) **!!CLOSED!!**
- **detection efficiency** (~fair sampling hypothesis)!

!! CLOSED!!

The entangled state

Photon

$$|\psi^-\rangle \cong \{|H\rangle_l \otimes |V\rangle_r - |V\rangle_l \otimes |H\rangle_r\}$$

$$P(H,\vec{n};H,\vec{m}) = P(V,\vec{n};V,\vec{m})$$
$$= \tfrac{1}{4}\left(1-\cos 2\phi_{nm}\right)$$

Kaon

$$|\psi^-\rangle \cong \left\{|K^0\rangle_l \otimes |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l \otimes |K^0\rangle_r\right\}$$

$$P(K^0,t_l;K^0,t_r) = P(\bar{K}^0,t_l;\bar{K}^0,t_r)$$
$$= \tfrac{1}{8}\Big(e^{-\Gamma_S t_l - \Gamma_L t_r} + e^{-\Gamma_L t_l - \Gamma_S t_r} - 2\cos(\Delta m \Delta t)\cdot e^{-\Gamma(t_l+t_r)}\Big)$$

No decay: $\Gamma_S = \Gamma_L = 0$

$$P(K^0,t_l;K^0,t_r) = P(\bar{K}^0,t_l;\bar{K}^0,t_r)$$
$$= \tfrac{1}{4}\left(1-\cos(\Delta m \Delta t)\right)$$

→ the time difference plays the same role as the angle in the photon case
→ for equal time measurements on both sides, we have the perfect EPR-like correlation

CHSH inequality for mesons

$$S(t_a,t_b,t_c,t_d) = \left|E(t_a;t_b) - E(t_a;t_c)\right| + \left|E(t_d;t_b) + E(t_d;t_c)\right| \leq 2$$

Quantum theory:

$$E^{QM}(t_a,t_b) = P(M^0 t_a, M^0 t_b) + P(\overline{M}^0 t_a, \overline{M}^0 t_b) - P(M^0 t_a, \overline{M}^0 t_b) - P(\overline{M}^0 t_a, M^0 t_b)$$
$$= -e^{-\frac{\Gamma_1+\Gamma_2}{2}(t_a+t_b)} \cdot \cos(\Delta m \Delta t)$$

K-mesons:

$S_{\max}^{Photon} = 2\sqrt{2} = 2.8$ **Violation!**

$S_{\max}^{Kaon} = 2$ **NO violation!**

$$x_{kaon} = \frac{2\Delta m}{\Gamma} = 0.946 \pm 0.003$$

B-mesons:

$S_{\max}^{Photon} = 2\sqrt{2} = 2.8$ **Violation!**

$S_{\max}^{B-meson} = 2$ **NO violation!**

$$x_{B-meson} = \frac{2\Delta m}{\Gamma} = 0.755 \pm 0.015$$

Violation?

PhysicsWeb - Mesons violate Bell's inequality - Microsoft Internet Explorer von Lycos Bertelsmann

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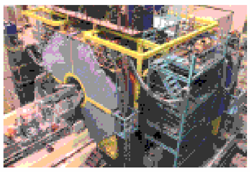
Mesons violate Bell's inequality

6 November 2003

The famous Bell's inequality of quantum mechanics has been tested in a high-energy particle physics experiment for the first time. The inequality was violated by three standard deviations in experiments with B mesons at the KEK laboratory in Japan - yet again confirming the predictions of quantum theory (arxiv.org/abs/quant-ph/0310192; *J. Mod. Optics* to be published). Previously most Bell's inequality experiments have been performed with photons or ions.

Experiments to test Bell's inequality involve measuring the properties of pairs of particles that are space-like separated in the sense of special relativity: in other words, there is no time for a light signal to travel between them within the duration of the experiment. In a typical Bell's inequality experiment the polarizations of a pair of photons are measured as the relative angle between the axes of polarizers making the measurements is varied.

Quantum mechanics predicts that "non-local" correlations can exist between the particles. This means that if one photon is polarized in, say, the vertical direction, the other will always be polarized in the horizontal direction, no matter how far away it is. However, some physicists argue that this cannot be true and that quantum particles must have local values - known as "hidden variables" - that we cannot measure.



Belle experiment

Bell and others showed that it was possible to distinguish between quantum mechanics and these hidden-variable theories in a certain type of experiment that measure a parameter known as S. Put simply, the local theories predict that S will always be less than two, whereas the quantum prediction is $S = 2\sqrt{2}$. When S is greater than two, Bell's inequality is said to be violated.

Apollo Go of the National Central University in Taiwan and co-workers in the Belle collaboration performed the experiment at the KEK B-factory. At this accelerator beams of electrons and positrons are collided to produce pairs of B mesons and their antiparticles, which then decay into lighter particles. The meson pairs behave like photon pairs, but instead of analyzing correlations between directions of polarization, the Belle team study particle-antiparticle correlations using a technique known as "flavour tagging". Go and colleagues calculated that S = 2.725, with error bars that mean that the inequality is violated by three standard deviations

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B mesons decay in a new way

Author
Peter Rodgers

Internet

Is there really a violation?

Normalization:

$$E^{QM}{}_{normalized}(t_a,t_b)=\frac{P(M^0 t_a,M^0 t_b)+P(\overline{M}^0 t_a,\overline{M}^0 t_b)-P(M^0 t_a,\overline{M}^0 t_b)-P(\overline{M}^0 t_a,M^0 t_b)}{\sum}$$

$$=-\frac{\cos(\Delta m\Delta t)}{\cosh(\Delta\Gamma\Delta t)}$$

B-mesons:

$$E^{QM}{}_{normalized}(t_a,t_b)=-\cos(\Delta m\Delta t)$$

- time evolution of mesons has to be unitary!!

$$\left|M_{1,2}(t)\right\rangle=e^{-i\lambda_{1,2}t}\left|M_{1,2}\right\rangle+\left|\Omega_{1,2}(t)\right\rangle$$

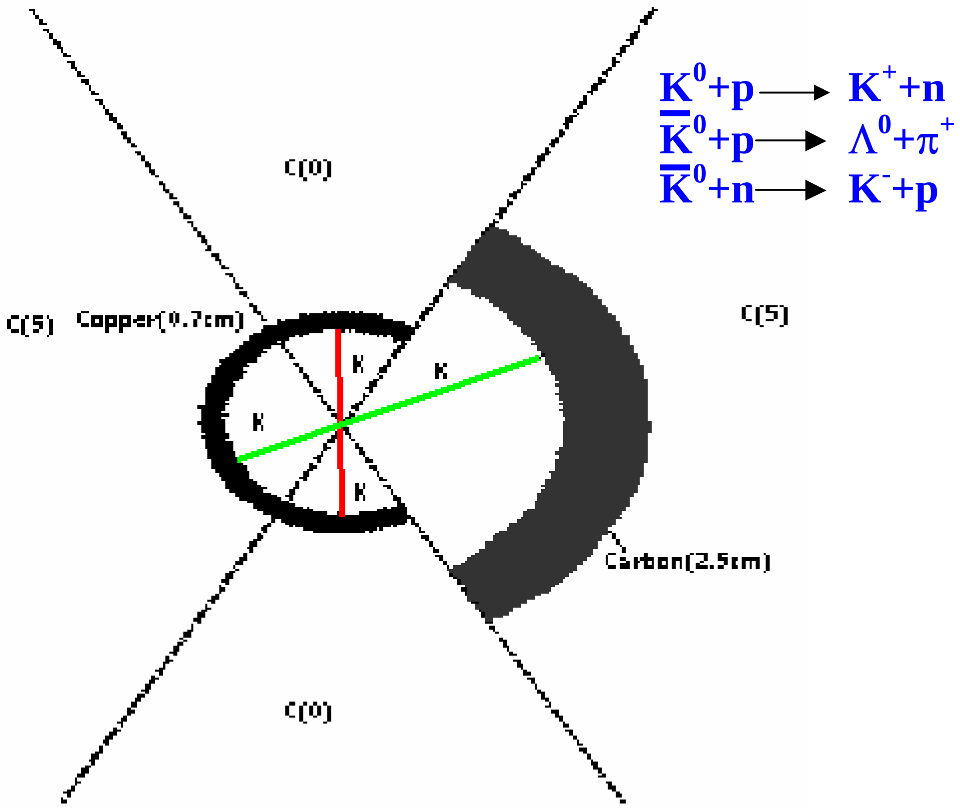
- leads to "different expectation value" (Are you a meson or not?)
- NO violation of CHSH inequality possible

Summary:

No violation if one considers the whole Hilbert space!
A violation can only be archived if one selects a very special sub-ensemble!

How to test entanglement?

CPLEAR-Experiment (1998)

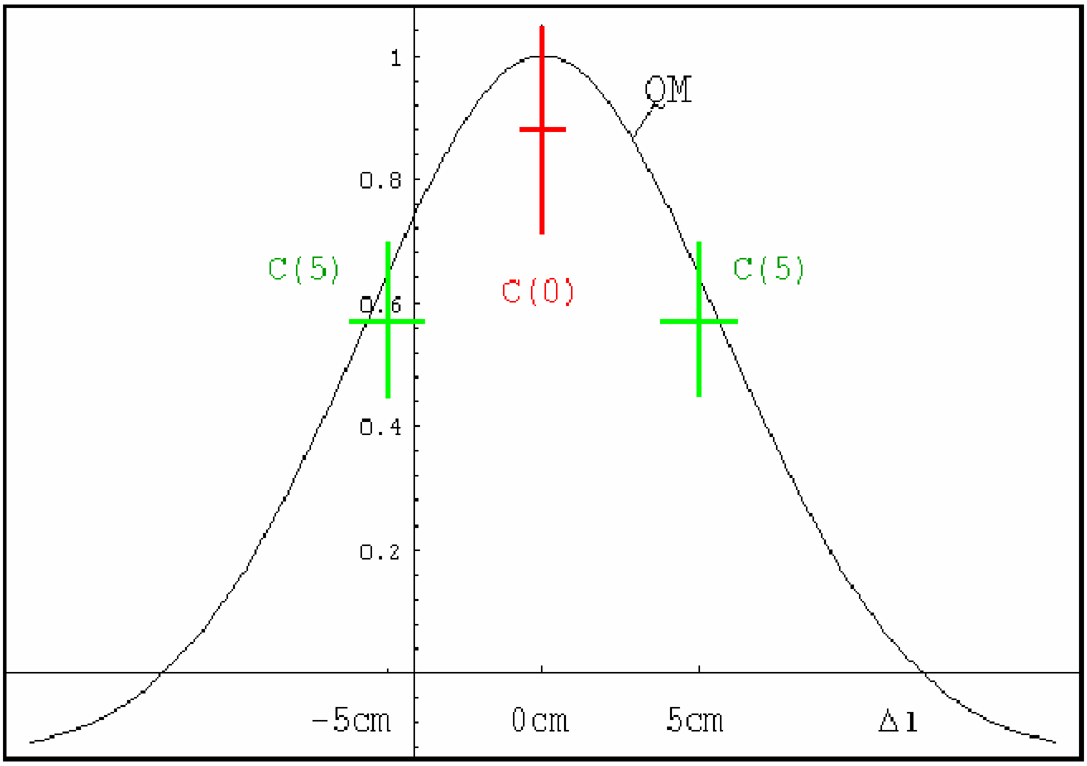


$$E^{QM}_{normalized}(t_a,t_b)=\frac{P(K^0t_a,K^0t_b)+P(\overline{K}^0t_a,\overline{K}^0t_b)-P(K^0t_a,\overline{K}^0t_b)-P(\overline{K}^0t_a,K^0t_b)}{\Sigma}$$

$$=-\frac{\cos(\Delta m\Delta t)}{\cosh(\Delta\Gamma\Delta t)}:=A^{QM}(t_a,t_b)$$

Experimental result:	Corrected theoretical result:
$A^{exp}(C(0))=0.81\pm0.17$	$A^{th}(C(0))=0.93$
$A^{exp}(C(5))=0.48\pm0.12$	$A^{th}(C(5))=0.56$

Is the produced state really entangled?



- How good do these two data points verify the quantum mechanical interference term?
- Is the Furry-Schrödinger hypothesis really ruled out?
- Is there decoherence in the system? Loss of entanglement?

A spontaneous factorisation of the wave function?

Schrödinger-Furry Hypothesis ($\zeta = 1$):

$$|\psi^-\rangle = |K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r$$

50% → $|K_S\rangle_l \otimes |K_L\rangle_r$

50% → $|K_L\rangle_l \otimes |K_S\rangle_r$

$$P^{QM}(K^0, t_r; K^0, t_l) = \| \underbrace{\langle K^0 | K_S(t_l)\rangle \langle K^0 | K_l(t_r)\rangle}_{A_1} - \underbrace{\langle K^0 | K_L(t_l)\rangle \langle K^0 | K_S(t_r)\rangle}_{A_2} \|^2$$

$$= |A_1|^2 + |A_2|^2 - 2\,\mathrm{Re}\{A_1^* A_2\}$$

$$P^{\zeta}(K^0, t_r; K^0, t_l) = |A_1|^2 + |A_2|^2 - 2\,\underbrace{(1-\zeta)}_{Modification}\,\mathrm{Re}\{A_1^* A_2\}$$

$$\Rightarrow \quad A^{\zeta}(\Delta t) = A^{QM}(\Delta t) \cdot (1-\zeta)$$

CPLEAR-experiment (1998): $\zeta = 0.13^{+0.16}_{-0.15}$

Bertlmann, Grimus and Hiesmayr, Phys. Rev. D, 60, 114032 (1999)

Factorisation into the strangeness basis

Schrödinger-Furry Hypothesis ($\zeta = 1$):

$$|\psi^-\rangle = |K^0\rangle_l \otimes |\overline{K}^0\rangle_r - |\overline{K}^0\rangle_l \otimes |K^0\rangle_r$$

50% → $|K^0\rangle_l \otimes |\overline{K}^0\rangle_r$

50% → $|\overline{K}^0\rangle_l \otimes |K^0\rangle_r$

$$P^{QM}(K^0,t_r;K^0,t_l) = \| \underbrace{\langle K^0 | K^0(t_l)\rangle \langle K^0 | \overline{K}^0(t_r)\rangle}_{A_1} - \underbrace{\langle K^0 | \overline{K}^0(t_l)\rangle \langle K^0 | K^0(t_r)\rangle}_{A_2} \|^2$$

$$= |A_1|^2 + |A_2|^2 - 2\,\mathrm{Re}\{A_1^* A_2\}$$

$$P^{\zeta}(K^0,t_r;K^0,t_l) = |A_1|^2 + |A_2|^2 - 2\underbrace{(1-\zeta)}_{Modification}\mathrm{Re}\{A_1^* A_2\}$$

$$A^{\zeta}_{K^0\overline{K}^0}(t_l,t_r) = \frac{\cos(\Delta m \Delta t) - \frac{1}{2}\zeta(\cos(\Delta m \Delta t) - \cos(\Delta m(t_l + t_r)))}{\cosh(\Delta\Gamma \Delta t) - \frac{1}{2}\zeta(\cosh(\Delta\Gamma \Delta t) - \cosh(\Delta\Gamma(t_l + t_r)))}$$

CPLEAR-experiment (1998): $\zeta_{K^0\overline{K}^0} = 0.41^{+0.67}_{-0.57}$

Bertlmann, Grimus and Hiesmayr, Phys. Rev. D, 60, 114032 (1999)

Spontaneous factorisation for B-mesons?

For dileptonic events ($B^0 \to l^+$; $\bar{B}^0 \to l^-$):

$$R = \frac{N_{B^0,B^0} + N_{\bar{B}^0,\bar{B}^0}}{N_{B^0,\bar{B}^0} + N_{\bar{B}^0,B^0}} = \frac{x^2 + \zeta \cdot \frac{Z(Basis)}{1+x^2}}{2 + x^2 - \zeta \cdot \frac{Z(Basis)}{1+x^2}}$$

$N_{B^0,B^0}, \ldots$ **time integrated numbers of like/unlike flavour events**

Experiments (CLEO, ARGUS):

$$\overline{R} = 0.189 \pm 0.044 \quad and \quad \overline{x} = 0.755 \pm 0.015$$

B_H, B_L basis ($Z=1+x^2$):

$$\zeta = \frac{R(2+x^2) - x^2}{1+R} \cdot \frac{1+x^2}{Z} \Longrightarrow \overline{\zeta} \pm \Delta\zeta = -0.0709 \pm 0.098$$

B^0, $\bar{B}^0$ basis ($Z=x^2$):

$$\Longrightarrow \overline{\zeta} \pm \Delta\zeta = -0.157 \pm 0.3$$

Bertlmann and Grimus, Found. Phys. 39, 426 (1997)
Bertlmann and Grimus, Phys. Rev. D 58, 34014 (1998)

How does the classical world appear out of the quantum world?

- **What distinguishes classical from quantum objects?**
- **Is there any kind of borderline?**
- **What is the precise structure of such a transition from quantum to classical?**

`Classical´ systems interact strongly with their natural environment → are not isolated → opens interpretation of classical properties within the framework of QT

Time-evolution of an open quantum system: according to Schrödinger-equation

$$\left|\Psi\right\rangle \otimes \left|\Phi\right\rangle \xrightarrow{t} \sum_{n,m} c_{nm} \left|\Psi_n\right\rangle \otimes \left|\Phi_m\right\rangle = \sum_n \sqrt{p_n(t)} \left|\tilde{\Psi}_n(t)\right\rangle \otimes \left|\tilde{\Phi}_n(t)\right\rangle$$

→ r.h.s. no single product in the general case
→ more than one component of the sum in the Schmidt representation is equivalent to the existence of quantum correlation

Decoherence …

- Is the irreversible formation of quantum correlations of a system with its environment
- Is a consequence of QT that affects virtually all physical systems
- Explains why "macroscopic" systems seem to posses classical properties
- Is a direct consequence of the Schrödinger equation, but has nonetheless been essentially overlooked for long time

The decoherence model

Liouville-von Neumann Eq.:

Decoherence

$$\frac{\partial \rho}{\partial t} = -iH\rho + i\rho H^t - D[\rho]$$

General form:

$$D[\rho] = \tfrac{1}{2}\sum_i (A_i^t A_i \rho + \rho A_i^t A_i - 2A_i \rho A_i^t)$$

1976: Lindblad;
Gorini-Kossakowski-Sudarshan

Choice of generators for the model:

$$A_i = \sqrt{\lambda}\, P_i = \sqrt{\lambda}\, |e_i\rangle\langle e_i| \qquad i = 1,2$$

$$|e_1\rangle = |K_S\rangle \otimes |K_L\rangle$$
$$|e_2\rangle = |K_L\rangle \otimes |K_S\rangle$$

$$D[\rho] = \lambda\left(P_1 \rho P_2 - P_2 \rho P_1\right)$$

$\lambda \geq 0$ **decoherence parameter**

- **Generates a completely positive map**
- **Conserves energy in case of $H=H^t$ since $[P_i,H]=0$**
- **Von Neumann entropy is not decreasing, because generators are Hermitian: $P_i=P^t_i$**

The solution for the decoherence model

$$\frac{\partial\rho}{\partial t} = -iH\rho + i\rho H - \lambda\left(P_1\rho P_2 - P_2\rho P_1\right)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left\{\underbrace{|K_S\rangle_l \otimes |K_L\rangle_r}_{} - \underbrace{|K_L\rangle_l \otimes |K_S\rangle_r}_{}\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{|e_1\rangle - |e_2\rangle\right\}$$

$$\rho(0) = |\Psi\rangle\langle\Psi| = \tfrac{1}{2}\left\{ |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - |e_1\rangle\langle e_2| - |e_2\rangle\langle e_1|\right\}$$

Solution:

$$\rho(t) = \sum \rho_{ij}(t)\,|e_i\rangle\langle e_j|$$

$$\rho_{11}(t) = \rho_{11}(0)\cdot e^{-2\Gamma t}$$
$$\rho_{22}(t) = \rho_{22}(0)\cdot e^{-2\Gamma t}$$
$$\rho_{12}(t) = \rho_{12}(0)\cdot e^{-2\Gamma t - \lambda t}$$

Bounds from experimental data

Time evolution:

$$\rho(t) = \tfrac{1}{2} e^{-2\Gamma t} \left\{ |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t} |e_1\rangle\langle e_2| - e^{-\lambda t} |e_2\rangle\langle e_1| \right\}$$

CPLEAR asymmetry term:

$$A^{QM}(t_l, t_r) = \frac{P(K^0, t_l; \overline{K}^0, t_l) - P(K^0, t_l; K^0, t_l)}{\Sigma} = \frac{\cos(\Delta m \Delta t)}{\cosh(\frac{\Delta\Gamma}{2}\Delta t)}$$

$$A^{\lambda}(t_l, t_r) = A^{QM}(\Delta t) \cdot e^{-\lambda \min\{t_l, t_r\}}$$

$$\zeta(t_l, t_r) = 1 - e^{-\lambda \min(t_l; t_r)}$$

$$A^{\zeta}(t_l, t_r) = A^{QM}(\Delta t) \cdot (1 - \zeta(t_l, t_r))$$

Data of CPLEAR experiment:

$$\overline{\lambda} = (1.84^{+2.50}_{-2.17}) \cdot 10^{-12}\ MeV$$

$$\overline{\Lambda} = \frac{\overline{\lambda}}{\Gamma_S} = 0.25^{+0.34}_{-0.32}$$

Bertlmann, Durstberger and Hiesmayr, Phys. Rev. A 68 (2003) 012111

B-meson system: Data of ARGUS, CLEO

$$\zeta(\lambda) = \frac{\lambda}{2\Gamma + \lambda}$$

$$\lambda_B = (-56 \pm 61) \cdot 10^{-12}\ MeV$$

$$\frac{\lambda}{\Gamma} = \frac{2\zeta}{(1-\zeta)}$$

$$\frac{\lambda_B}{\Gamma} = -0.13 \pm 0.14$$

Bertlmann and Grimus, Phys. Rev. D 64, 056004 (2001)

Discussion of the decoherence model

1. Von Neumann entropy S
2. Entanglement of formation E /Concurrence C
3. Lack of separability (not included)
4. Generalized Bell inequality-Tetrahedron (not included)

Von Neumann entropy S:

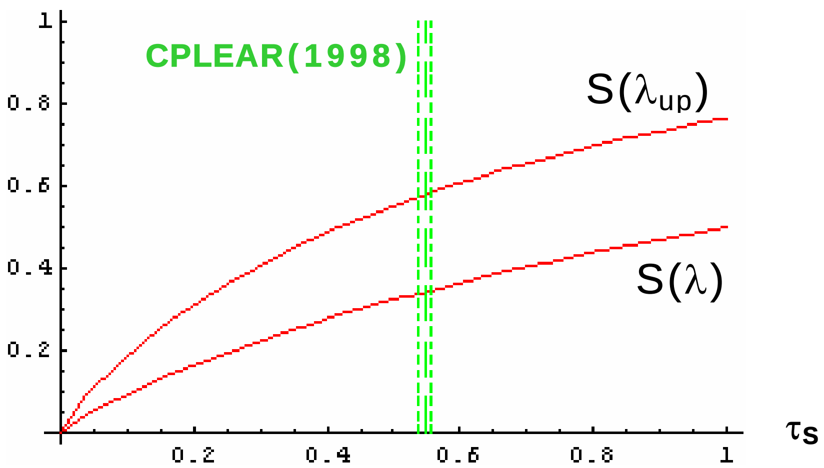
How much uncertainty is in the system?

$$\rho(t) = \tfrac{1}{2} e^{-2\Gamma t} \left\{ |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t} |e_1\rangle\langle e_2| - e^{-\lambda t} |e_2\rangle\langle e_1| \right\}$$

Normalizing: $\rho_N(t) = \dfrac{\rho(t)}{Tr\{\rho(t)\}}$

System:

$$S(\rho_N(t)) = -Tr\{\rho_N(t) \log_2 \rho_N(t)\}$$
$$= -\frac{1-e^{-\lambda t}}{2} \log_2 \frac{1-e^{-\lambda t}}{2} - \frac{1+e^{-\lambda t}}{2} \log_2 \frac{1+e^{-\lambda t}}{2}$$

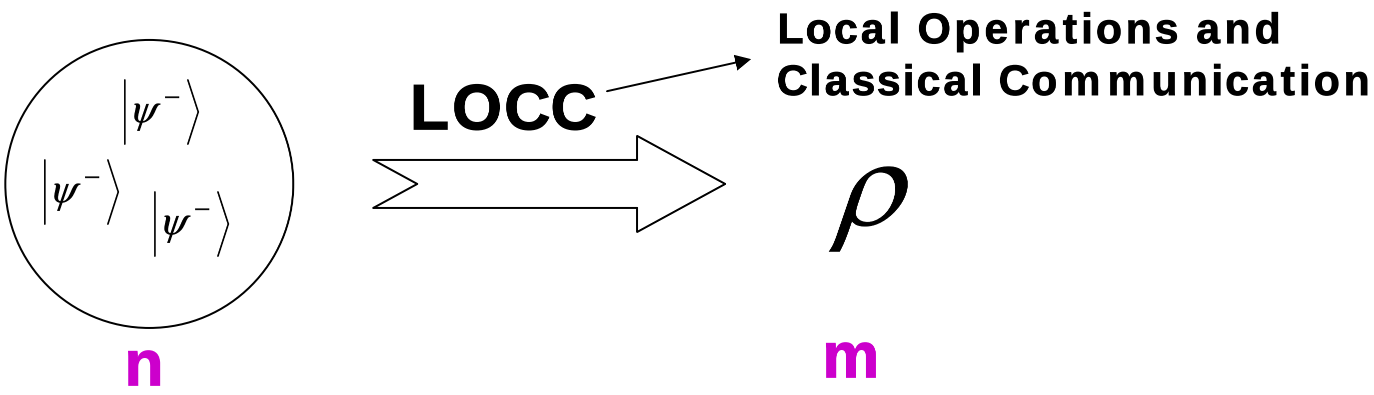


Subsystem:

$$S(Tr_r \rho_N(t)) = 1 \qquad \forall\ t \geq 0$$

Entanglement of Formation / Concurrence

How much resources are needed to create a given state?



$$\rho = \sum p_i \rho_i = \sum p_i |\psi_i\rangle\langle\psi_i|$$

Entanglement of formation

$$E(\rho) = \min \sum p_i S(Tr_l(\rho_i)) \quad \sim n/m$$

Bennett et al. (1996)

$$= \mathcal{E}(C(\rho))$$

Concurrence

Wootters, Hill (1997)

Measures of entanglement

S: How much uncertainty?

E: How much resources are needed to create a given state?

Loss of Concurrence:

$$1 - C(\rho_N(t)) = 1 - e^{-\lambda t} = \zeta(t)$$

Loss of entanglement:

$$1 - E(\rho_N(t)) = \frac{1}{\ln 2}\zeta(t) = \frac{\lambda}{\ln 2} t$$