Hadronic mass moments in $B \to X_c \ell \nu$

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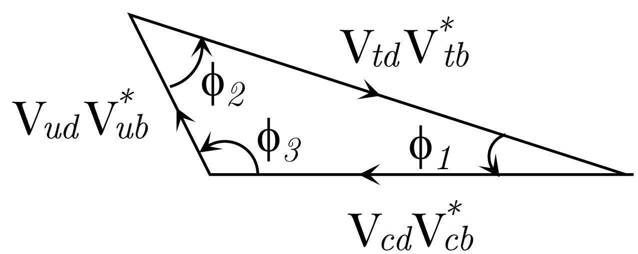
- Outline of the talk
  - theoretical motivation
  - experimental procedure
  - results and systematic uncertainty
  - cross-checks
  - summary and outlook

the quark mixing matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

the unitarity triangle



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Theoretical motivation

- $1/m_b$ expansion of $\Gamma(B \to X_c l \nu)$ (Heavy Quark Expansion)

$$\Gamma(B \to X_c l \nu) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left(P_0 + \frac{1}{m_b} P_1(\bar{\Lambda}) + \frac{1}{m_b^2} P_2(\bar{\Lambda}, \lambda_1, \lambda_2) + \mathcal{O}(\frac{1}{m_b^3}) \right)$$

- P_n are *calculable* polynomial functions (actually, expansions in α_s) of the *non-perturbative* parameters $\bar{\Lambda}$, λ_1 and λ_2

- $\bar{\Lambda}$, λ_1 and λ_2 can be interpreted intuitively
  - $\bar{\Lambda}$: energy of the light quark and gluon degrees of freedom
    $m_B = m_b + \bar{\Lambda} + \mathcal{O}(\frac{1}{m_b})$
  - $-\lambda_1$: average momentum-squared of the b-quark inside the hadron
  - λ_2/m_b: energy of the hyperfine interaction of the b-quark spin with the light degrees of freedom, can be determined from spectroscopy

Theoretical motivation (2)

- A similar $1/m_b$ expansion containing *the same parameters* $\bar{\Lambda}$, λ_1 and λ_2 exists for other inclusive observables like leptonic or hadronic mass moments
- $\bar{\Lambda}$ and λ_1 can thus be measured by experiments
- precision measurement of $|V_{cb}|$
  - ultimate precision around 2%
  - $B^0 \to D^{*-} l^+ \nu$ method limited by lattice error (5%)
- parton-hadron duality
  - underlying assumption of Heavy Quark Expansion
  - some authors claim that it is a sizeable source of theoretical uncertainty
  - by measuring $\bar{\Lambda}$ and λ_1 with different methods, this assumption can be tested

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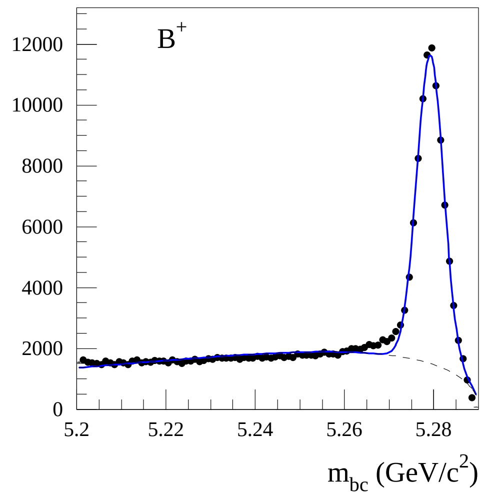
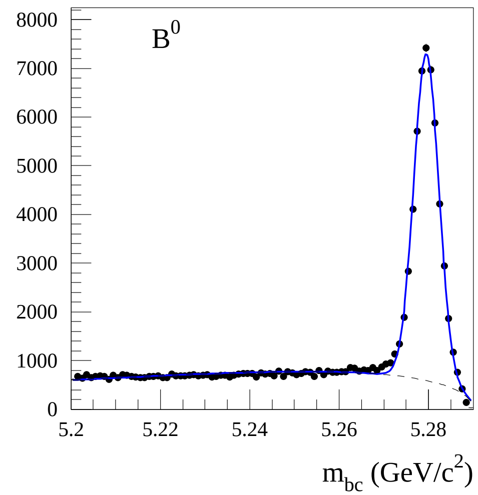
Experimental procedure

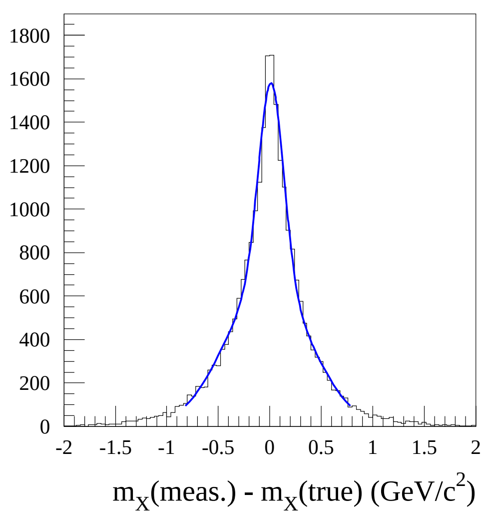
- 140 fb^{-1} of $\Upsilon(4S)$ data
- fully reconstruct the tag-side B meson
  $B \to D^{(*)}\pi$, $D^{(*)}\rho$ and $D^{(*)}a_1$

$N(B^+) = 76,155 \pm 511$



$N(B^0) = 46,863 \pm 374$



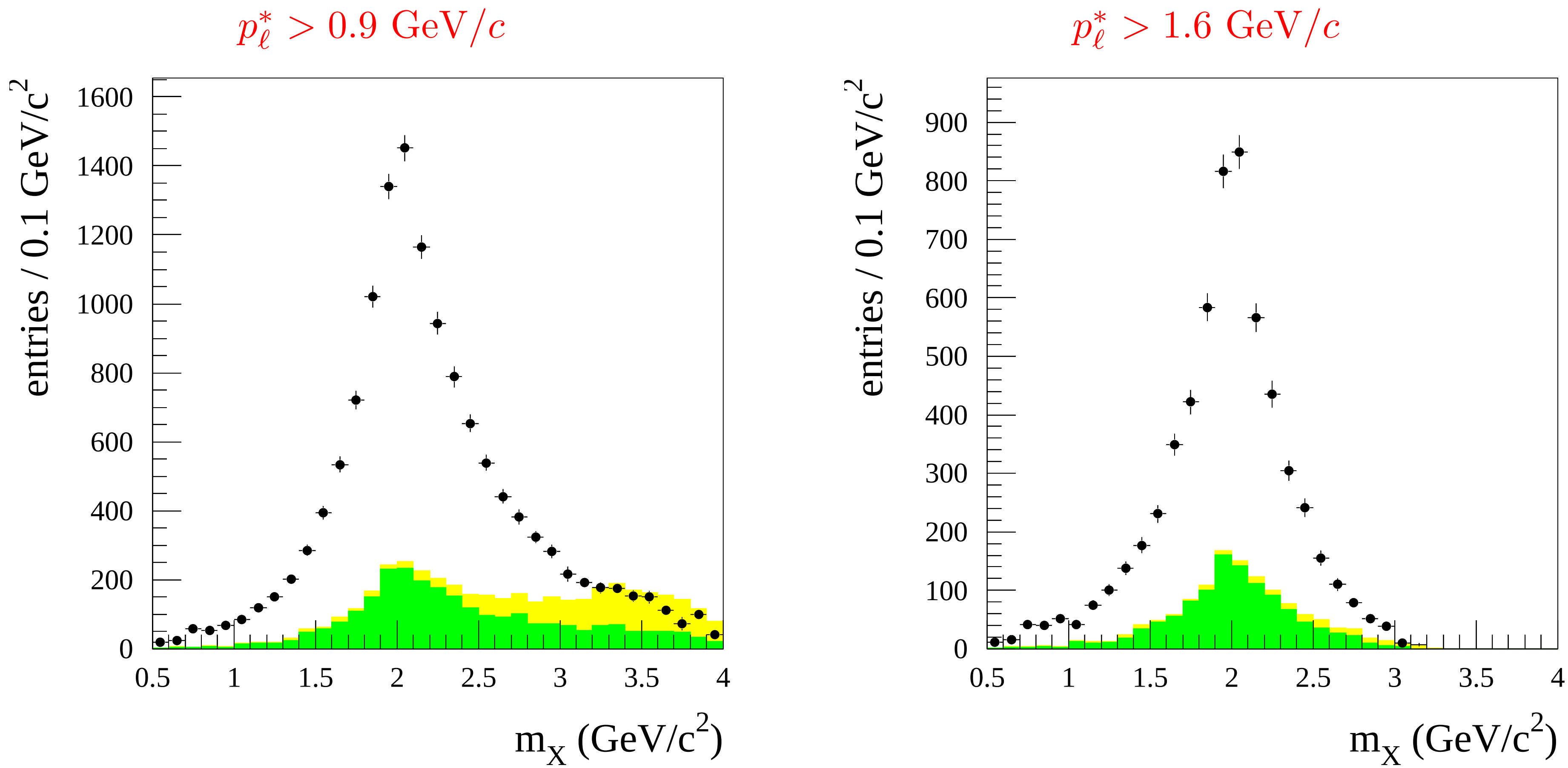


$\sigma \approx 120\ \mathrm{MeV}/c^2$

- identify a lepton (electron or muon) on the signal-side
  - choose events with exactly one lepton
  - for B^+ tags: $Q_\ell \cdot Q_B > 0$
  - $|m^2_{miss}| < 2\ \mathrm{GeV}/c^2$
- reconstruct the X system from the remaining particles

$$p_X = p_{HER} + p_{LER} - p_{B_{tag}} - p_\ell - p_\nu,$$
$$p_\nu = (|\vec{p}_{miss}|, \vec{p}_{miss})$$

m_X distribution



● continuum subtracted real data

(yellow) background from secondary or fake leptons

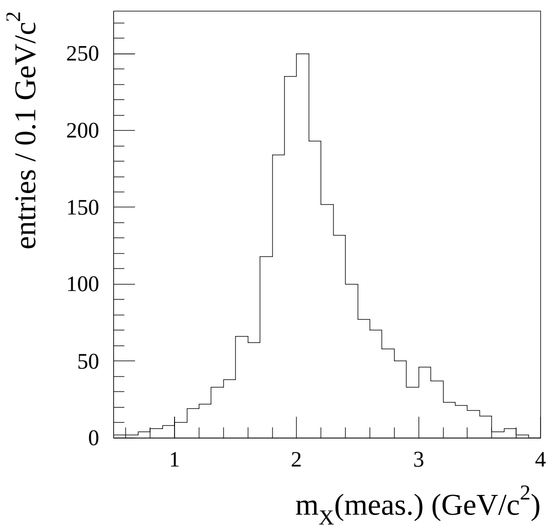
(green) combinatorial full reconstruction background

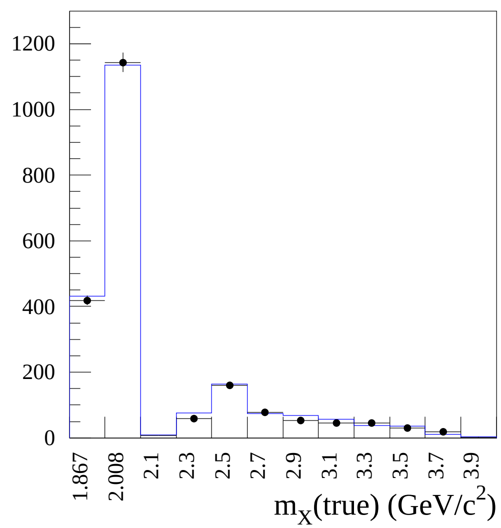
Experimental procedure (2)

- unfold the measured m_X and m_X^2 distributions
- using the Singular Value Decomposition (SVD) algorithm

  A. Höcker, V. Kartvelishvili, Nucl. Instr. Meth. **A 372**, 469 (1996)
- and calculate the mean values, $\langle m_X \rangle$ and $\langle m_X^2 \rangle$, of the unfolded distributions

validation of the unfolding procedure on Monte Carlo events



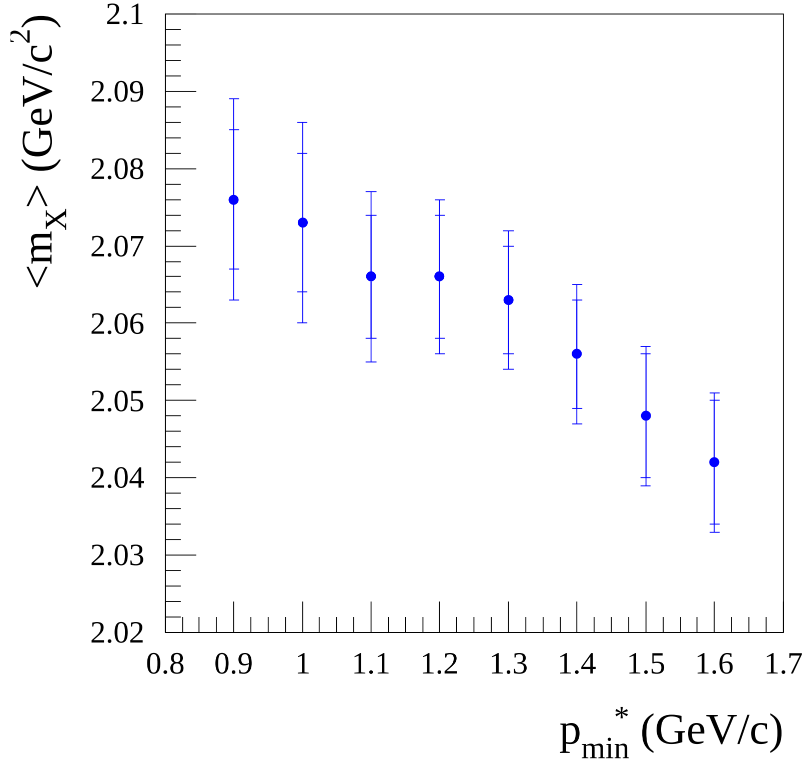


for a set of Monte Carlo events:

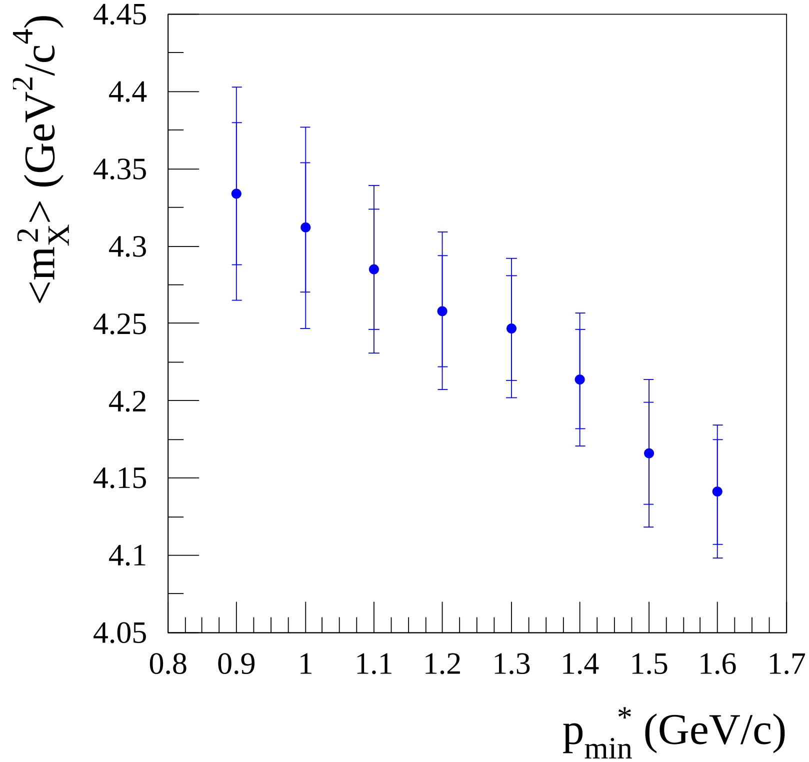
- true,
  histogram, right-hand side
- measured, and
  histogram, left-hand side
- unfolded m_X spectrum
  data points, right-hand side

Results for different lepton momentum thresholds

$\langle m_X \rangle$



$\langle m_X^2 \rangle$



preliminary

p^*_{min} (GeV/c)	$\langle m_X \rangle$ (GeV/c^2)	detector / bkgrd.	unfolding	X_c model
0.9	$2.076 \pm 0.009 \pm 0.010$	0.006	0.008	0.003
1.0	$2.073 \pm 0.009 \pm 0.010$	0.005	0.008	0.003
1.1	$2.066 \pm 0.008 \pm 0.008$	0.005	0.005	0.003
1.2	$2.066 \pm 0.008 \pm 0.006$	0.004	0.004	0.003
1.3	$2.063 \pm 0.007 \pm 0.006$	0.004	0.005	0.003
1.4	$2.056 \pm 0.007 \pm 0.006$	0.003	0.004	0.003
1.5	$2.048 \pm 0.008 \pm 0.005$	0.003	0.003	0.002
1.6	$2.042 \pm 0.008 \pm 0.004$	0.003	0.001	0.002

p^*_{min} (GeV/c)	$\langle m_X^2 \rangle$ (GeV^2/c^4)	detector / bkgrd.	unfolding	X_c model
0.9	$4.334 \pm 0.046 \pm 0.051$	0.041	0.027	0.014
1.0	$4.312 \pm 0.042 \pm 0.049$	0.037	0.030	0.013
1.1	$4.285 \pm 0.039 \pm 0.037$	0.032	0.017	0.012
1.2	$4.258 \pm 0.036 \pm 0.036$	0.027	0.021	0.012
1.3	$4.247 \pm 0.034 \pm 0.030$	0.023	0.016	0.011
1.4	$4.214 \pm 0.032 \pm 0.028$	0.024	0.009	0.011
1.5	$4.166 \pm 0.033 \pm 0.035$	0.022	0.025	0.011
1.6	$4.141 \pm 0.034 \pm 0.027$	0.022	0.010	0.013

preliminary

Systematic error

- detector and background modeling error estimated from
  - consistency between different sub-samples (B^+ electron, B^+ muon, B^0 electron, B^0 muon)
  - variation of cuts (B_{tag} signal window, m^2_{miss} cut)
  - variation of background normalization
- unfolding error estimated from
  - variation of the eigenvalue components in the unfolding solution
- X_c model error estimated by
  - varying the fraction of $B \to D^* \ell \nu$, $B \to D \ell \nu$ and $B \to D^{**}/D^{(*)} \pi \ell \nu$ within $\pm 10\%$, $\pm 10\%$ and $\pm 30\%$, respectively

Correlations between different measurements

- strong correlations due to overlapping data samples
- more than 90% for neighbouring points, 66% for extreme cut values
- correlation coefficients can be estimated from amount of overlap

$$\mathrm{cor}(w_i, w_j) = \frac{c_{ij}}{\sqrt{n_i}\sqrt{n_j}}$$

n_i (n_j) ...events in sample i (j)
c_{ij} ...overlap between i and j

[Kish, Survey Sampling, p. 458]

p^*_{min} (GeV/c)		$\langle m_X \rangle$								$\langle m_X^2 \rangle$						
		0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\langle m_X \rangle$	0.9	1.000	0.963	0.924	0.884	0.840	0.788	0.730	0.661	0.984	0.954	0.920	0.882	0.838	0.787	0.729
	1.0		1.000	0.960	0.919	0.873	0.819	0.759	0.687	0.961	0.991	0.955	0.916	0.871	0.818	0.757
	1.1			1.000	0.957	0.909	0.853	0.790	0.715	0.930	0.959	0.995	0.954	0.907	0.852	0.789
	1.2				1.000	0.950	0.891	0.826	0.748	0.893	0.922	0.956	0.997	0.948	0.890	0.824
	1.3					1.000	0.938	0.869	0.787	0.850	0.877	0.910	0.949	0.998	0.937	0.868
	1.4						1.000	0.926	0.839	0.798	0.824	0.855	0.891	0.937	0.998	0.924
	1.5							1.000	0.905	0.740	0.763	0.792	0.825	0.868	0.925	0.998
	1.6								1.000	0.670	0.691	0.717	0.747	0.786	0.837	0.904
$\langle m_X^2 \rangle$	0.9									1.000	0.969	0.934	0.896	0.852	0.800	0.741
	1.0										1.000	0.964	0.924	0.879	0.825	0.765
	1.1											1.000	0.959	0.912	0.856	0.793
	1.2												1.000	0.951	0.893	0.827
	1.3													1.000	0.939	0.870
	1.4														1.000	0.927
	1.5															1.000
	1.6															

correlation coefficients including statistical uncertainty only

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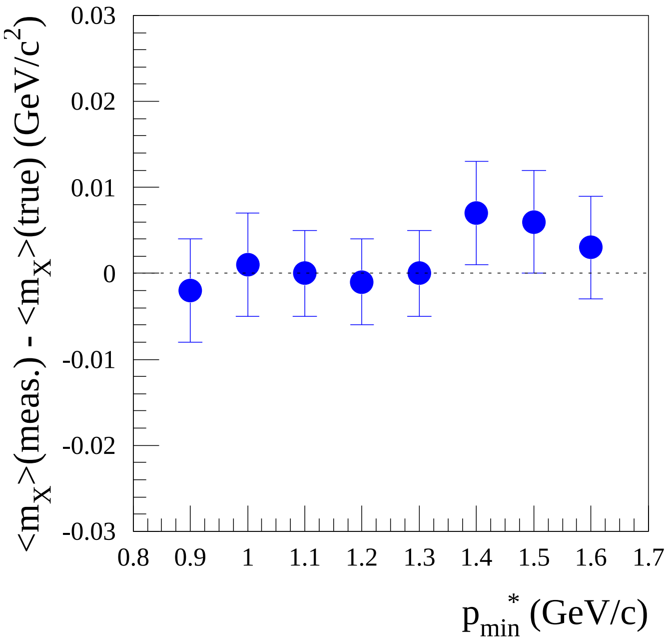
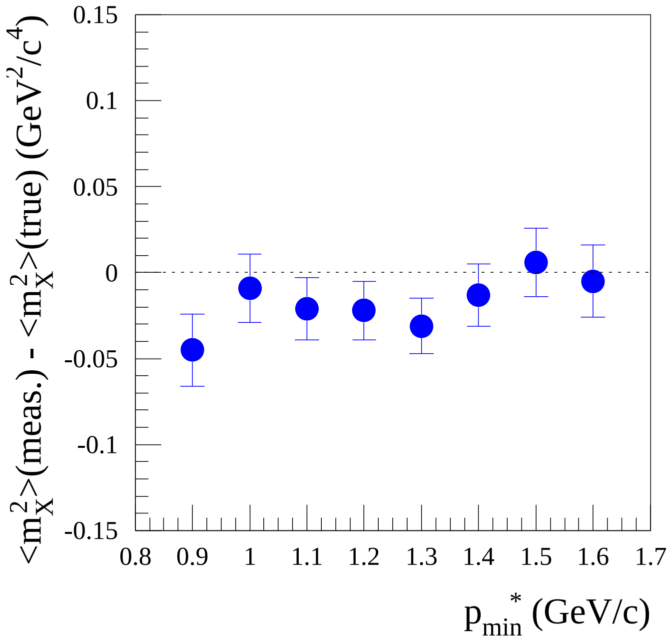
Cross-check #1

- divide available Monte Carlo into two sets
- use first set to calculate the unfolding matrix
- measure $\langle m_X \rangle$ and $\langle m_X^2 \rangle$ for the other set, compare to the true value

$\langle m_X \rangle$



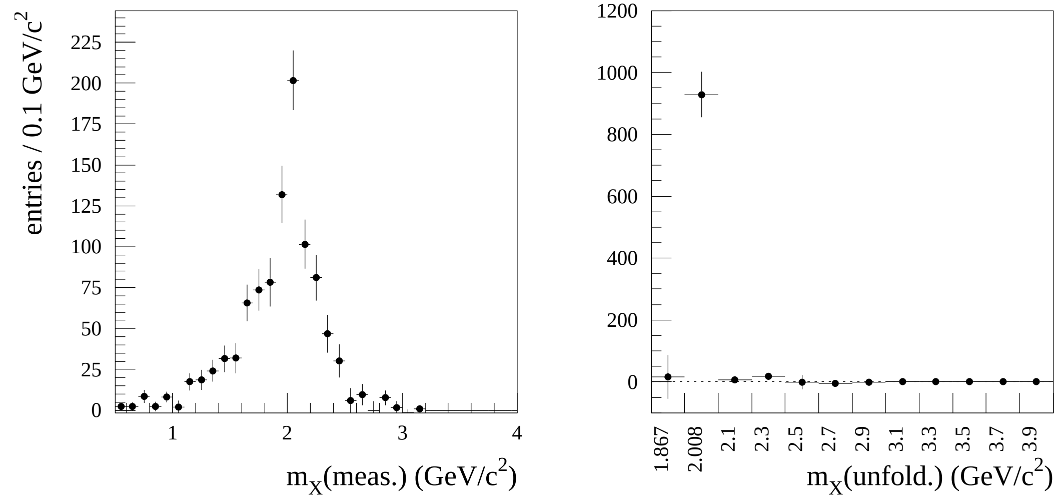
$\langle m_X^2 \rangle$



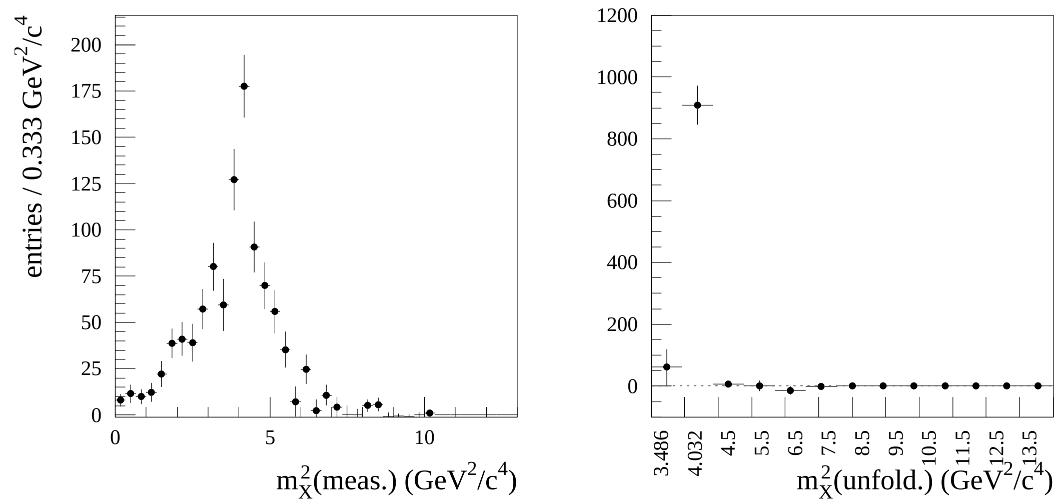
Cross-check #2

- measure m_X and m_X^2 distributions in the data
- subtract fake/secondary lepton and full reconstruction background
- subtract the $B \to D\ell\nu$ and $B \to D^{**}/D^{(*)}\pi\ell\nu$ components using Monte Carlo
- apply unfolding – the remaining distribution should only contain $B \to D^*\ell\nu$

m_X unfolding



m_X^2 unfolding



Summary and outlook

- The first and second hadronic mass moments in $B \to X_c \ell \nu$, $\langle m_X \rangle$ and $\langle m_X^2 \rangle$, have been measured for $p_\ell^* > 0.9, \ldots, 1.6$ GeV/c
- The results are compatible with recent measurements of CLEO and BABAR
- This result has been shown at the summer conferences [hep-ex/0408139], and we plan to publish soon
- There are still many things to do from the experimental side
  - improve background subtraction
  - PHOTOS correction
- Are there also suggestions from theory side?
  - higher moments
  - other observables (moments around spin averaged D mass)
  - different lepton momentum regions (reduce correlations)