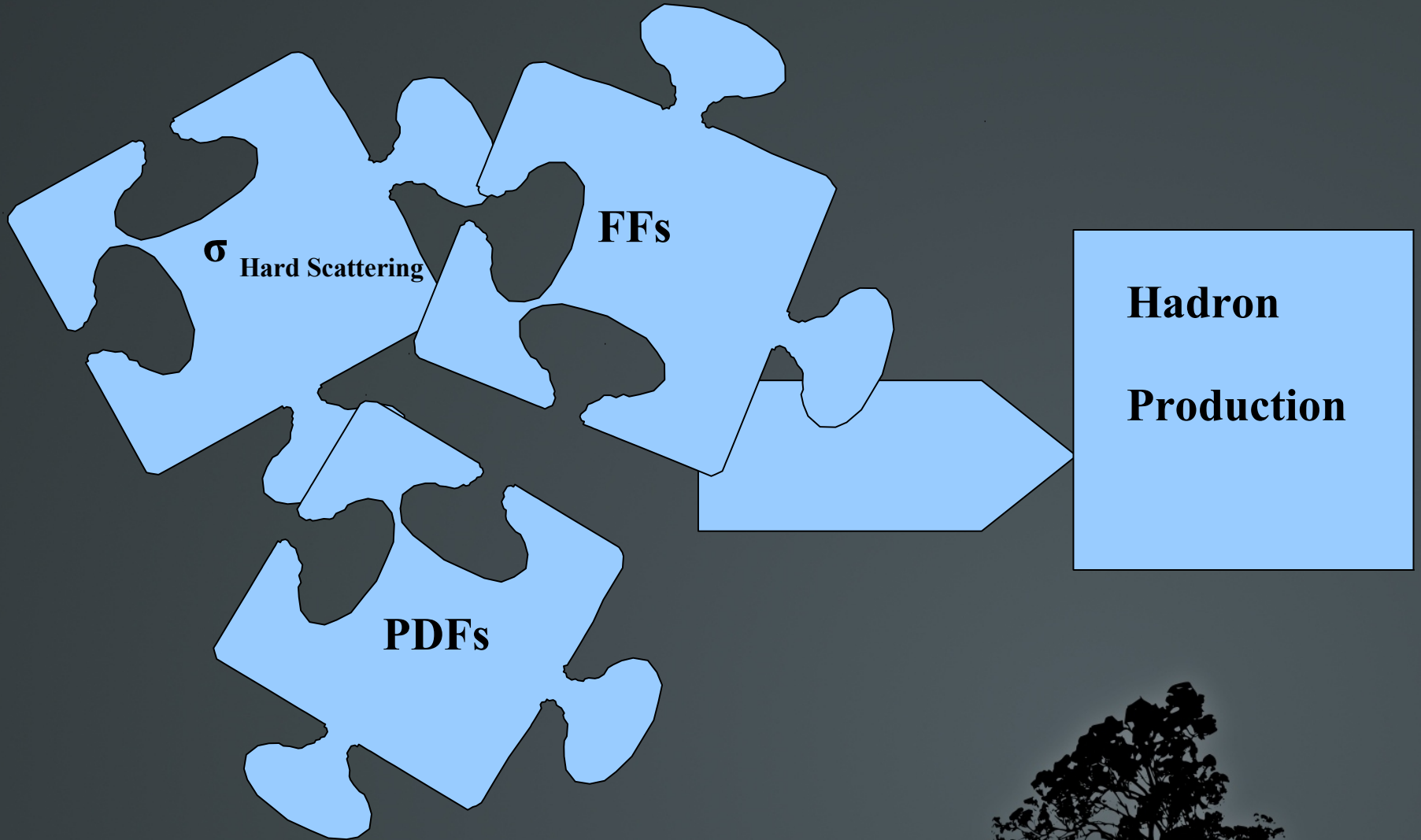


QCD Analysis
and Calculation of
Fragmentation Functions from
Hadron Multiplicities

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Hadronization



Introduction and Motivation

- Parton Distribution Functions (PDFs): Initial Particle
- Fragmentation Functions (FFs): Final State in a scattering Process
- Non-perturbative components need data input
- These functions are „very sensitive to data“
- Detailed look at nucleon substructure
- Most significant contribution to calculation of fragmentation functions and parton distribution functions: Single-inclusive annihilation: $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow h^{+,-,0} X$
- Advantages: process independence, insight in hadron structure and proton spin
- Disadvantages: difficulty entangling favored from unfavored FFs



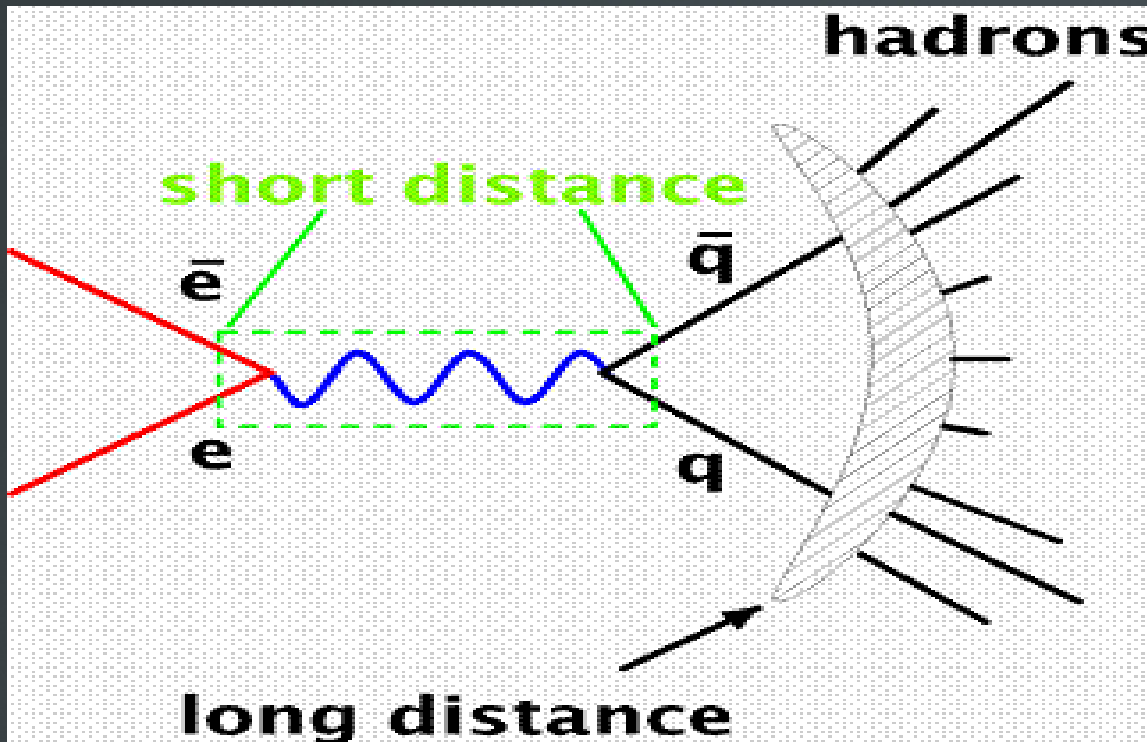
QCD Framework

- Cross sections (DIS and SIA) decomposed into convolutions between perturbatively calculated components and two non perturbative, universal components: FFs and PDFs
- Perturbatively calculated: Coefficient Functions and Splitting Functions
- Non-Perturbative component: Fragmentation Functions



Fragmentation

- e^+e^- annihilation: $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow h^{+,-,0} X$
- Fragmentation function: probability distribution that a parton at a short distance $1/Q$ fragments into a hadron with fraction z of the parent momentum k



$e^+ + e^-$ - Annihilation

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^H}{dz} = \mathcal{F}^H(z, Q^2) = \sum_q \hat{e}_q^2 \frac{\sigma_0}{\pi} [2F_1^H(z, Q^2) + F_L^H(z, Q^2)] \quad (2.1)$$

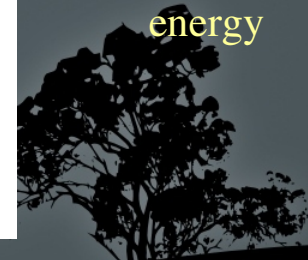
$$\sigma_{tot} = \sum_q \hat{e}_q^2 \sigma_0 \left[1 + \frac{\alpha_S(Q^2)}{\pi} \right] \quad (2.2)$$

$$\sigma_0 = \frac{4\pi\alpha_S^2(Q^2)}{Q^2}$$

$$\begin{aligned} \mathcal{F}^H(z, Q^2) &= \sum_i C_i(z, Q^2) \otimes D_i^H(z, Q^2) \\ &= \sum_{i=\text{partons}} \int_z^1 \frac{d\zeta}{\zeta} C_i(\zeta, Q^2) D_i^H\left(\frac{z}{\zeta}, Q^2\right) \end{aligned} \quad (2.3)$$

$$\begin{aligned} 2F_1^H(z, Q^2) &= \sum_q \hat{e}_q^2 \{ [D_q^H(z, Q^2) + D_{\bar{q}}^H(z, Q^2)] + \\ &\quad + \frac{\alpha_S(Q^2)}{2\pi} [C_q^1 \otimes (D_q^H + D_{\bar{q}}^H) + C_g^1 \otimes D_g^H](z, Q^2) \} \end{aligned} \quad (2.4)$$

- 2.1: cross section at c.m.s energy
- 2.2: total hadronic cross section
- α_S :_running coupling constant
- $z = 2 E_H/Q$
- $\sqrt{s}/2 = Q/2 =$ beam energy
- Convolutions with:
- C_{ij} : Coefficient functions:
probability of creating a parton i with a momentum fraction of the beam energy



Renormalization Group

- Dealing with infinities in a theory
- Scale dependence of QCD coupling is defined by β -function (possible negativity of β -function leads to asymptotic freedom)
- Modified Minimal Subtraction Scheme (MS-Scheme)
- Investigate parameters of a theory at different energies
- Running coupling constant α_s
- Renormalization Group Equation (Eq. 2.12)

$$2\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 \quad (2.12)$$

$$\beta_0 = 11 - \frac{2f}{3} \quad (2.13)$$

$$\beta_1 = 51 - \frac{19f}{3} \quad (2.14)$$

$$\beta_2 = 2857 - \frac{5033f}{9} + \frac{325f^2}{27} \quad (2.15)$$



Asymptotic Freedom and Running Coupling Constant

- Negative Beta-Function
- Renormalized QCD coupling decreases with high energies
- Scale dependence of QCD coupling is defined by β -function
- Approximate solution for $Q^2 > m_c$:

$$\frac{\alpha_s(Q^2)}{4\pi} \approx \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)} - \frac{\beta_1}{\beta_0^3} \frac{\ln \ln\left(\frac{Q^2}{\Lambda^2}\right)}{\left(\ln\left(\frac{Q^2}{\Lambda^2}\right)\right)^2}$$

$$\Lambda_{\overline{\text{MS}}} = 0.217^{+25}_{-23} \text{ GeV}$$



DGLAP Evolution

- Fit flexible parametrization to data (acc.t. DSS):

$$D_i^H(z, Q^2) = \frac{N_i z^{\alpha_i} (1-z)^{\beta_i} [1 + \gamma_i (1-z)^{\delta_i}]}{B [2 + \alpha_i, \beta_i + 1] + \gamma_i B [2 + \alpha_i, \beta_i + \delta_i + 1]} \quad (4.2)$$

- Use fragmentation function at different cms energies
- Evolution with increasing energy scale: DGLAP Evolution equation

$$\frac{\partial}{\partial \ln Q} D_i^H(z, Q^2) = \sum_j \int_z^1 \frac{dx}{x} \frac{\alpha_S}{2\pi} P_{ji}(x, \alpha_S) D_j^H\left(\frac{z}{x}, Q^2\right) \quad (2.8)$$

- With D_j^H being the fragmentation function of the final parton, P_{ij} being splitting functions and D_i^H parametrization for the FF
- Splitting functions: probability for finding a parton i coming from a parton j with a certain fraction of the parent momentum



DGLAP Evolution

- In LO P_{ij} in e^+e^- the same as in DIS
- Same probability for emitting a gluon, regardless of flavor
- Equal probability for gluon creating a quark-antiquark pair for all flavors
- P_{ij} s LO: (in Mellin Space)

B. Mele, P. Nason / Heavy quarks in QCD

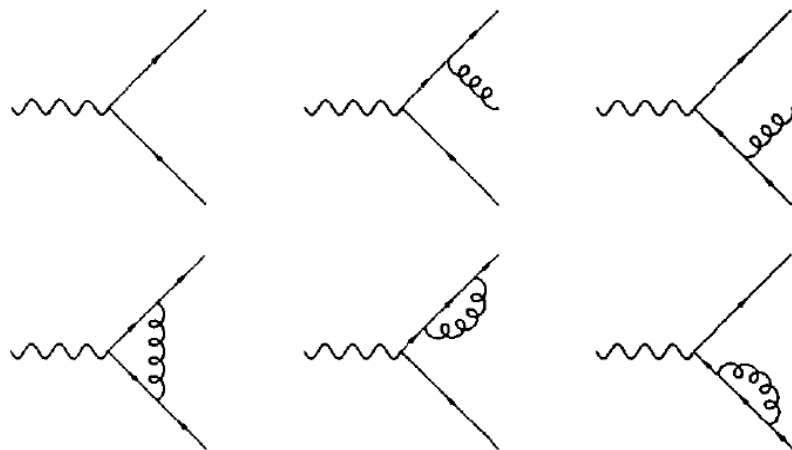


Fig. 2. The diagrams contributing to the fragmentation function to order α_s .

$$P_{qq}(j) = C_F \left(-\frac{1}{2} + \frac{1}{j(j+1)} - S_2(j) \right) \quad (\text{A.1})$$

$$P_{qg}(j) = T_F \left(\frac{(2+j+j^2)}{j(j+1)(j+2)} \right) \quad (\text{A.2})$$

$$P_{gq}(j) = C_F \left(\frac{(2+j+j^2)}{j(j^2-1)} \right) \quad (\text{A.3})$$

$$P_{gg}(j) = 2C_A \left(-\frac{1}{12} + \frac{1}{j(j-1)} + \frac{1}{(j+1)(j+2)} - S_1(j) \right) \quad (\text{A.4})$$

Mellin Space

- DGLAP Evolution: convolution of D_i^H and P_{ij} s
- Cross section: convolution of D_j^H and C_{ij} s
- Numerically very long and difficult calculations
- Transformation into Mellin Space: convolutions become multiplications
- Mellin Transform:

$$D_{jM}^H(n, Q^2) = \int_0^1 dz z^{n-1} D_j^H(z, Q^2) \quad (2.10)$$

- Disadvantage: all complex values of j need to be known for inversion



Mellin Space

Inverse Mellin Transform:

$$D_j^H(z, Q^2) = \frac{1}{2\pi i} \int_C dn z^{-n} D_{jM}^H(n, Q^2) \quad (2.11)$$

- Contour c has to be right of rightmost singularity
- C can be tilted by Φ

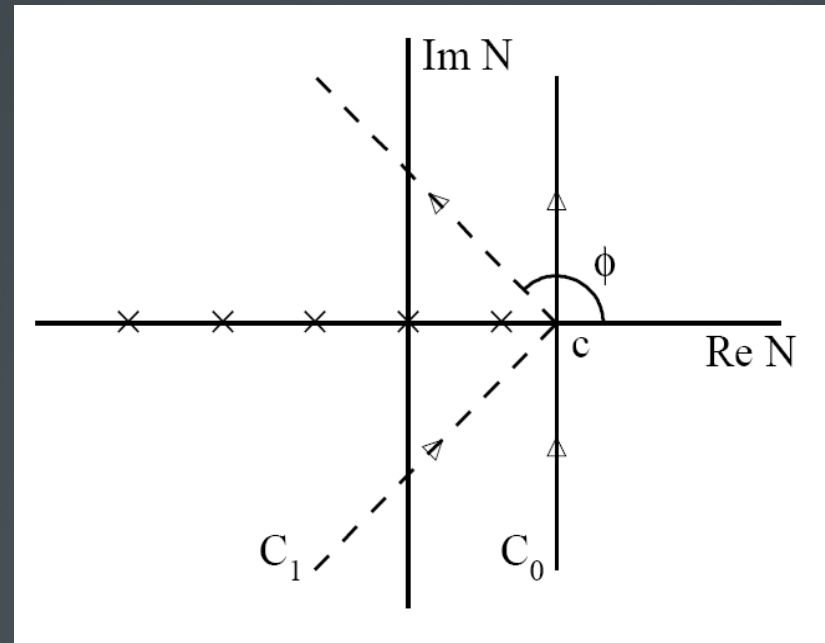
$$a(z) = \frac{1}{2\pi i} \int_0^\infty z^{-j} a(j) dj$$

w.:

$$j = c + x * \exp[i * \phi]$$

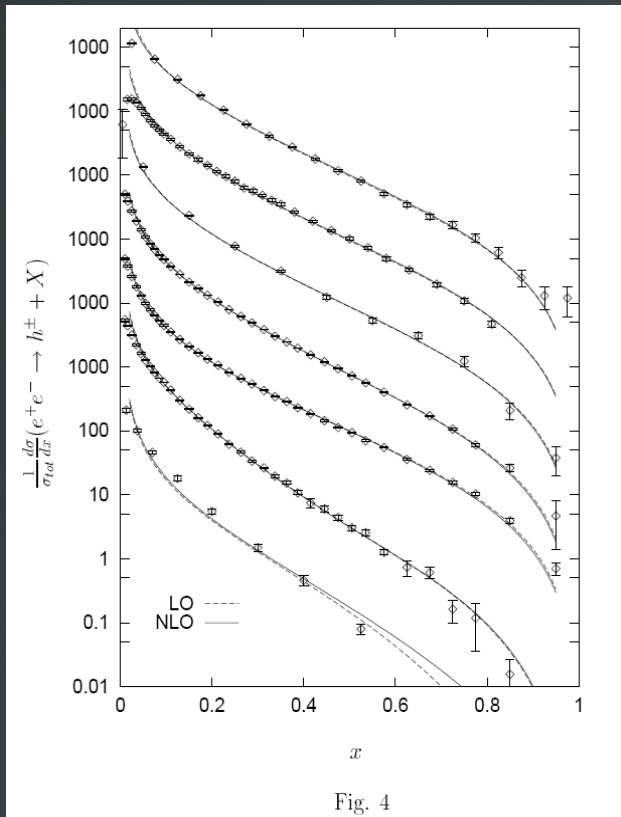
$$\phi = \frac{3}{4} * \text{Pi}$$

$$c = 1.3;$$



Data

- SIA data from OPAL: $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow h + X$

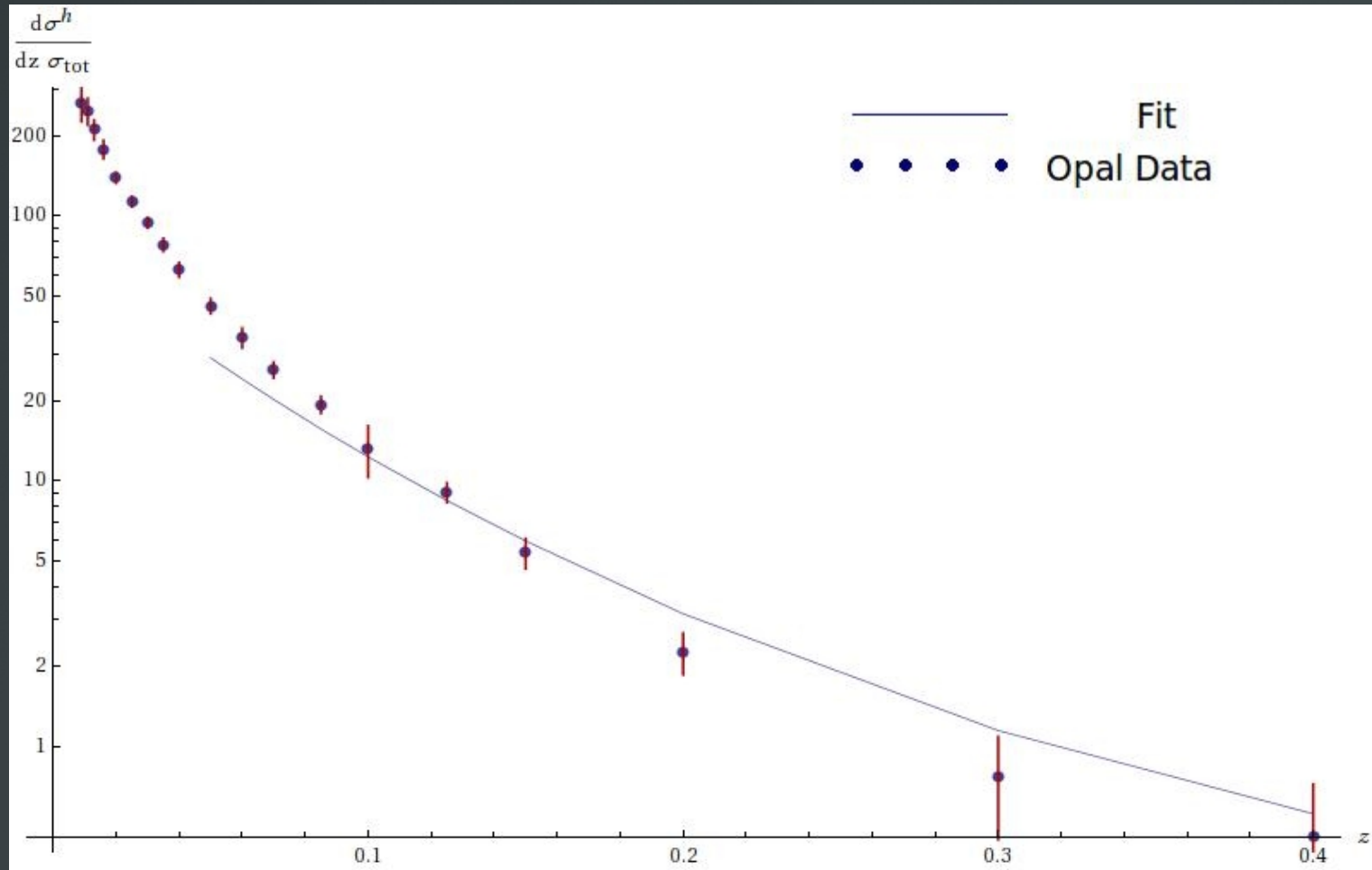


- Example from OPAL (91.5 GeV)
- Theoretical calculations done by DSS compared with data
- Differential cross section of inclusive hadron production in LO (dashed line)
- Differential cross section if inclusive hadron production in NLO (solid line)
- Lower 4 curves for Q of 91.5 GeV

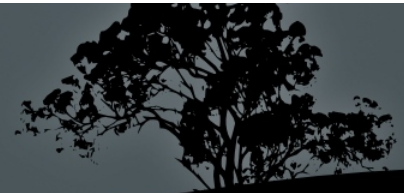
- SIA Data from Belle: $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow h + X$
- Energy: 10.52 GeV



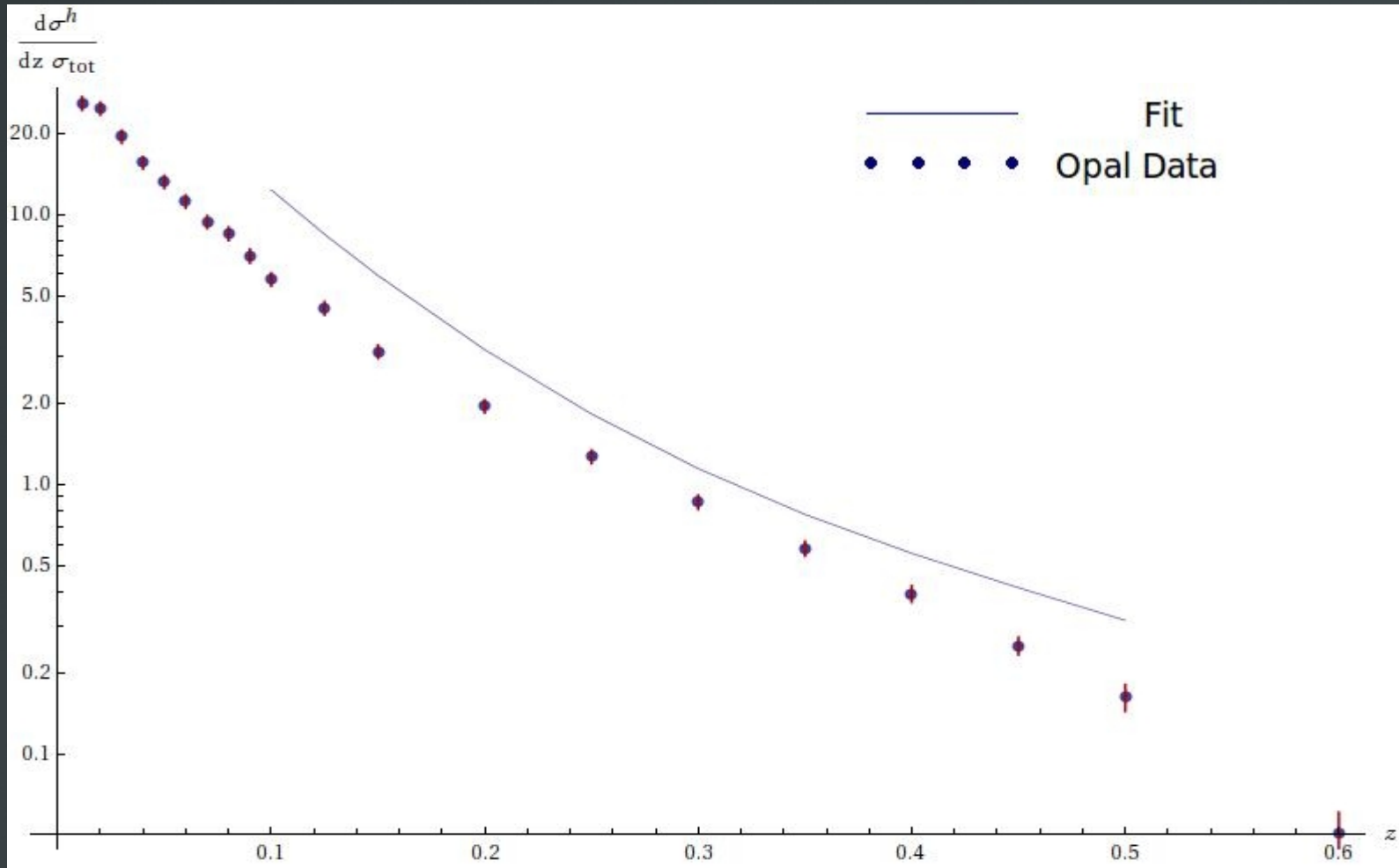
Results: LO Fit



Pi0 vs OPAL Data

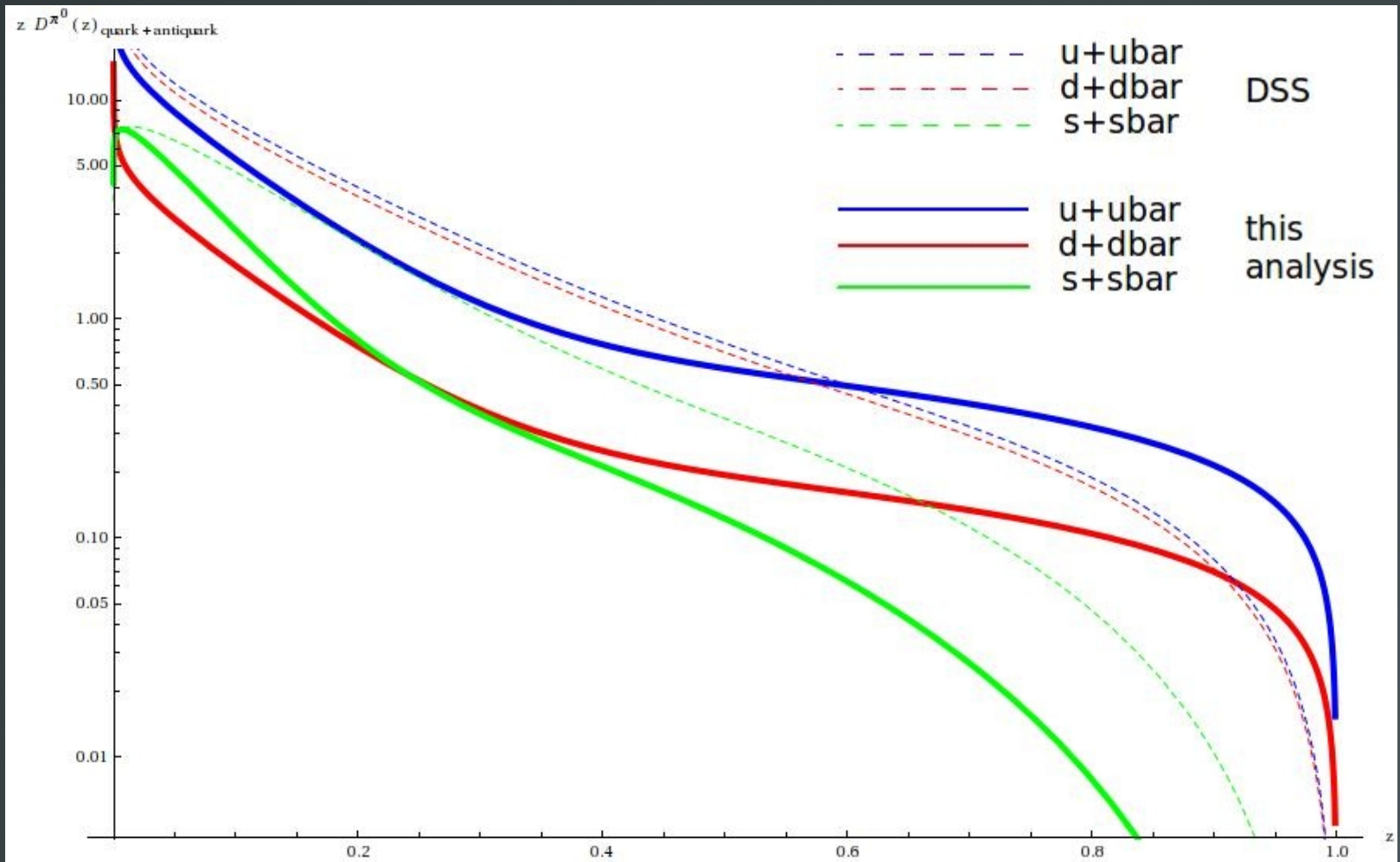


Results: LO Fit



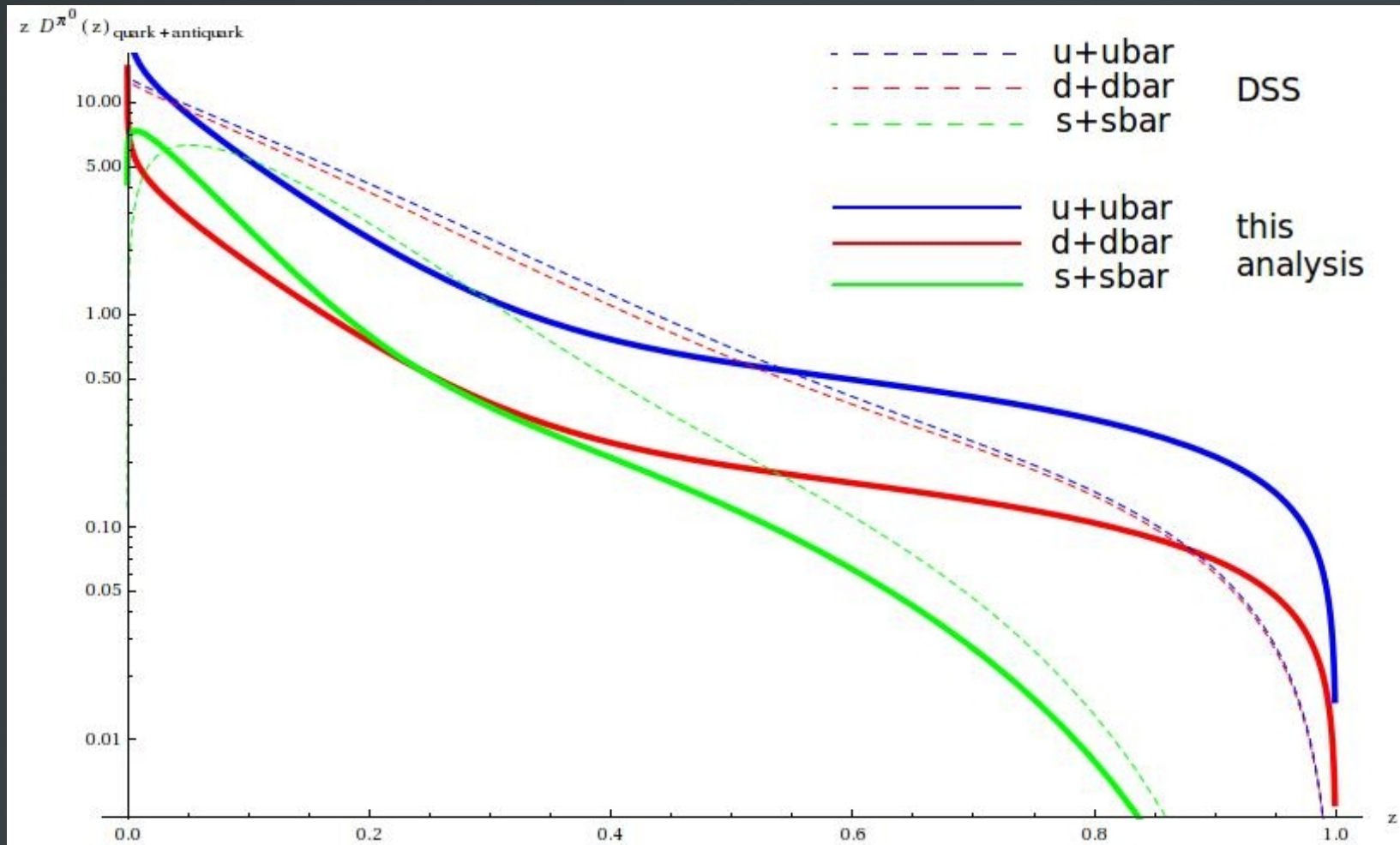
K0 vs OPAL Data

Results: Pion-FFs f. light quarks



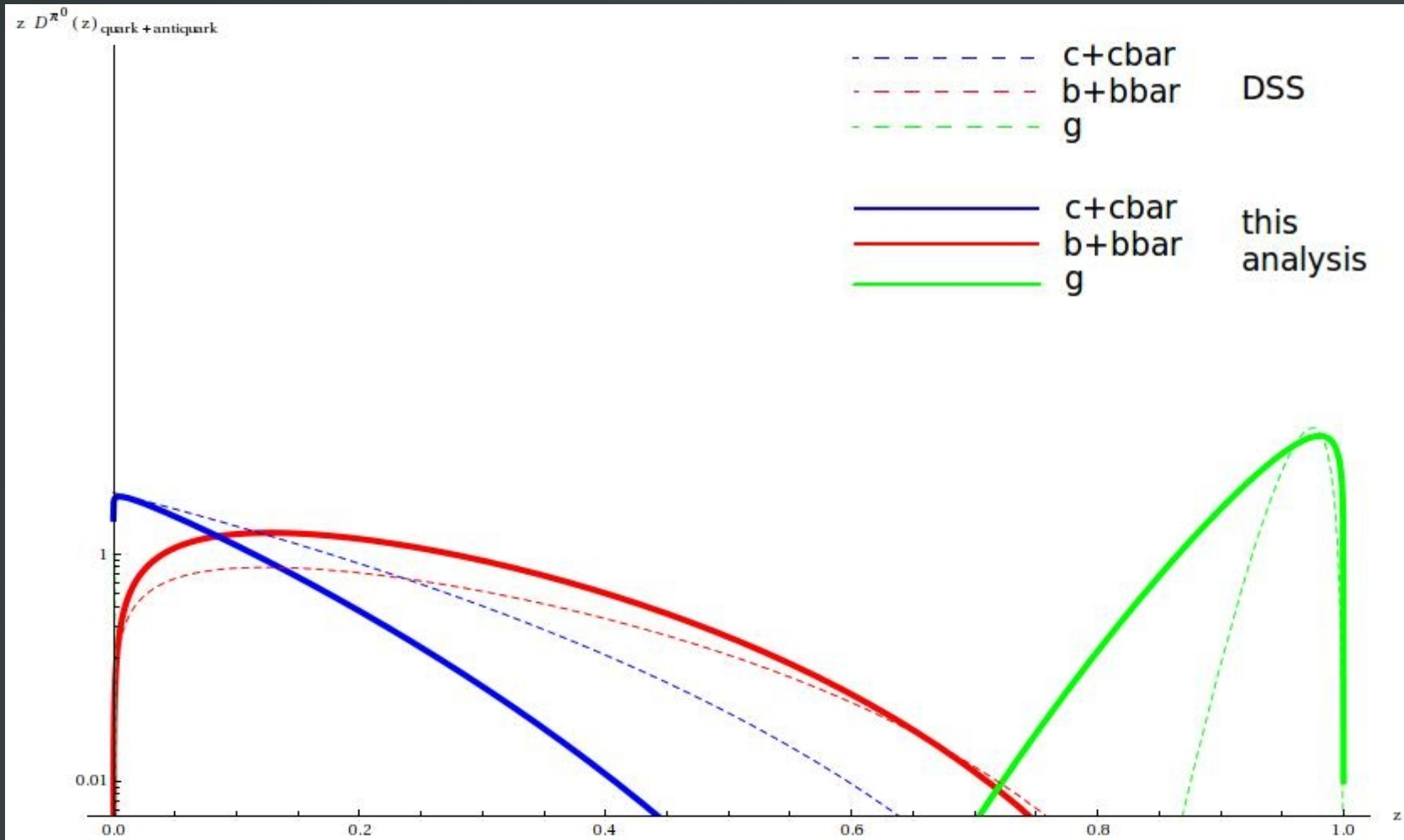
Pi0 – uds : LO vs. LO by DSS

Results: Pion-FFs f. light quarks



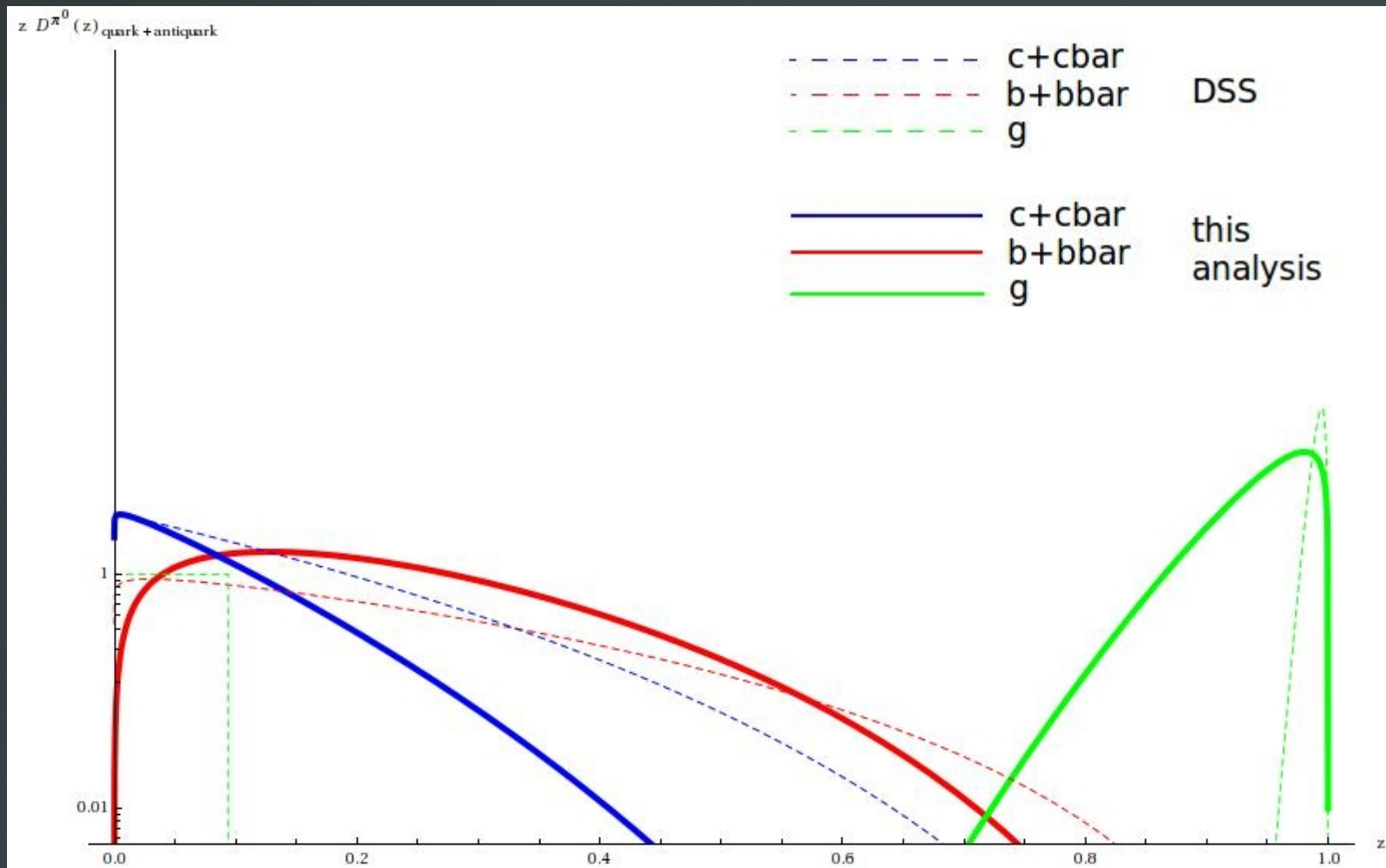
Pi0 - uds : LO vs. NLO by DSS

Results: Pion-FFs heavy quarks and gluon



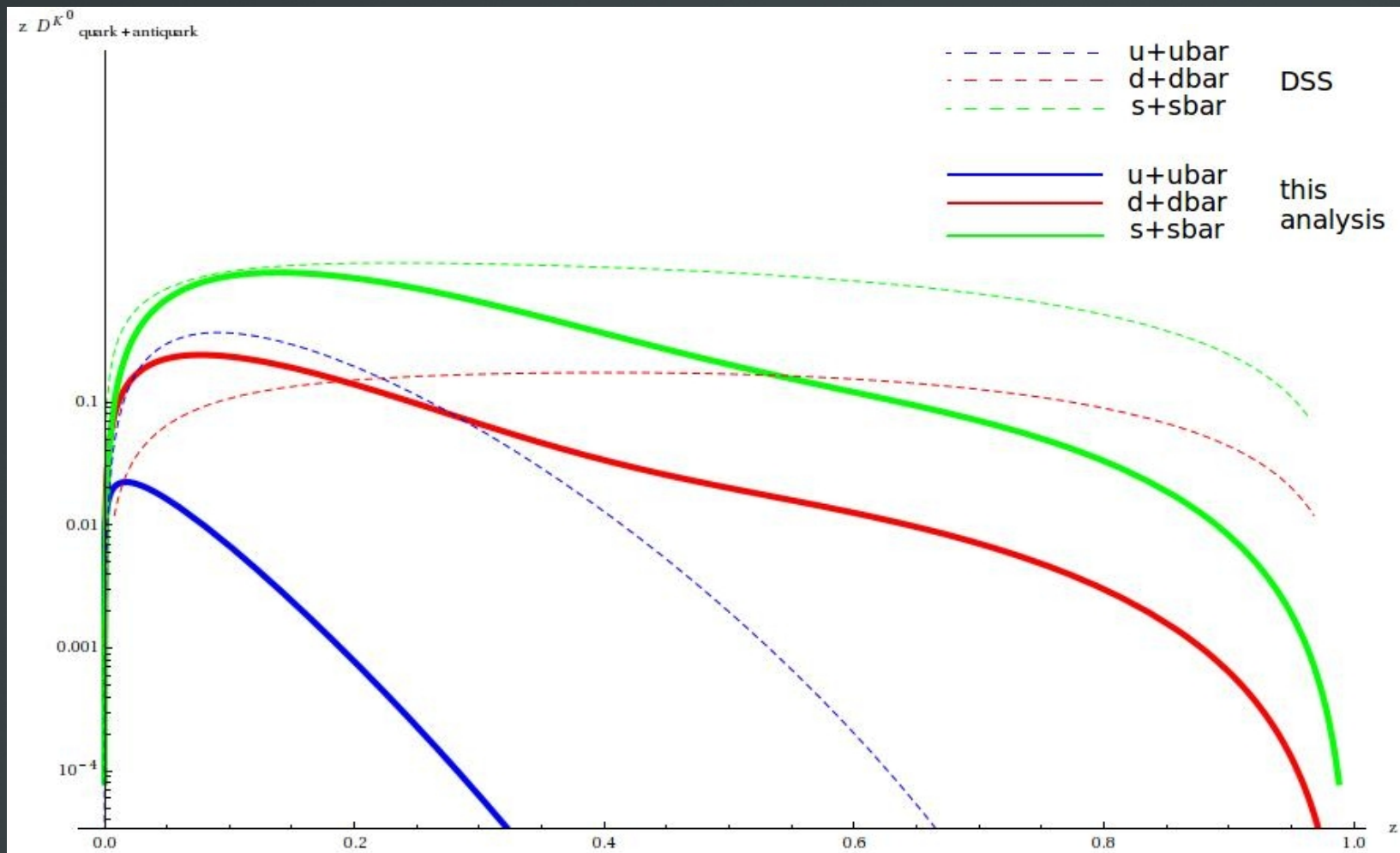
Pi0 – cbg : LO vs. LO by DSS

Results: Pion-FFs heavy f. quarks and gluon



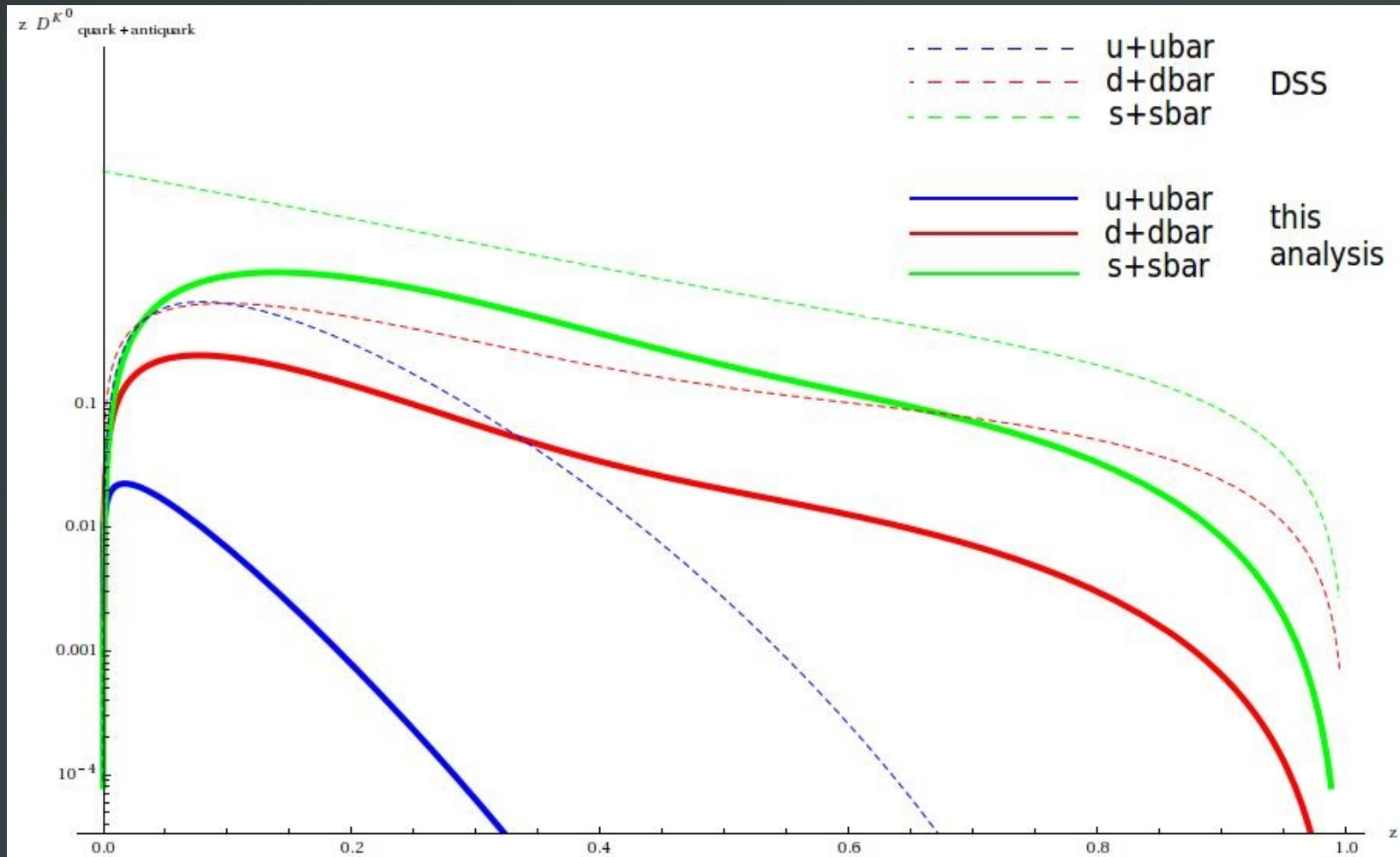
Pi0 - cbg : LO vs. NLO by DSS

Results: Kaon-FFs f. light quarks



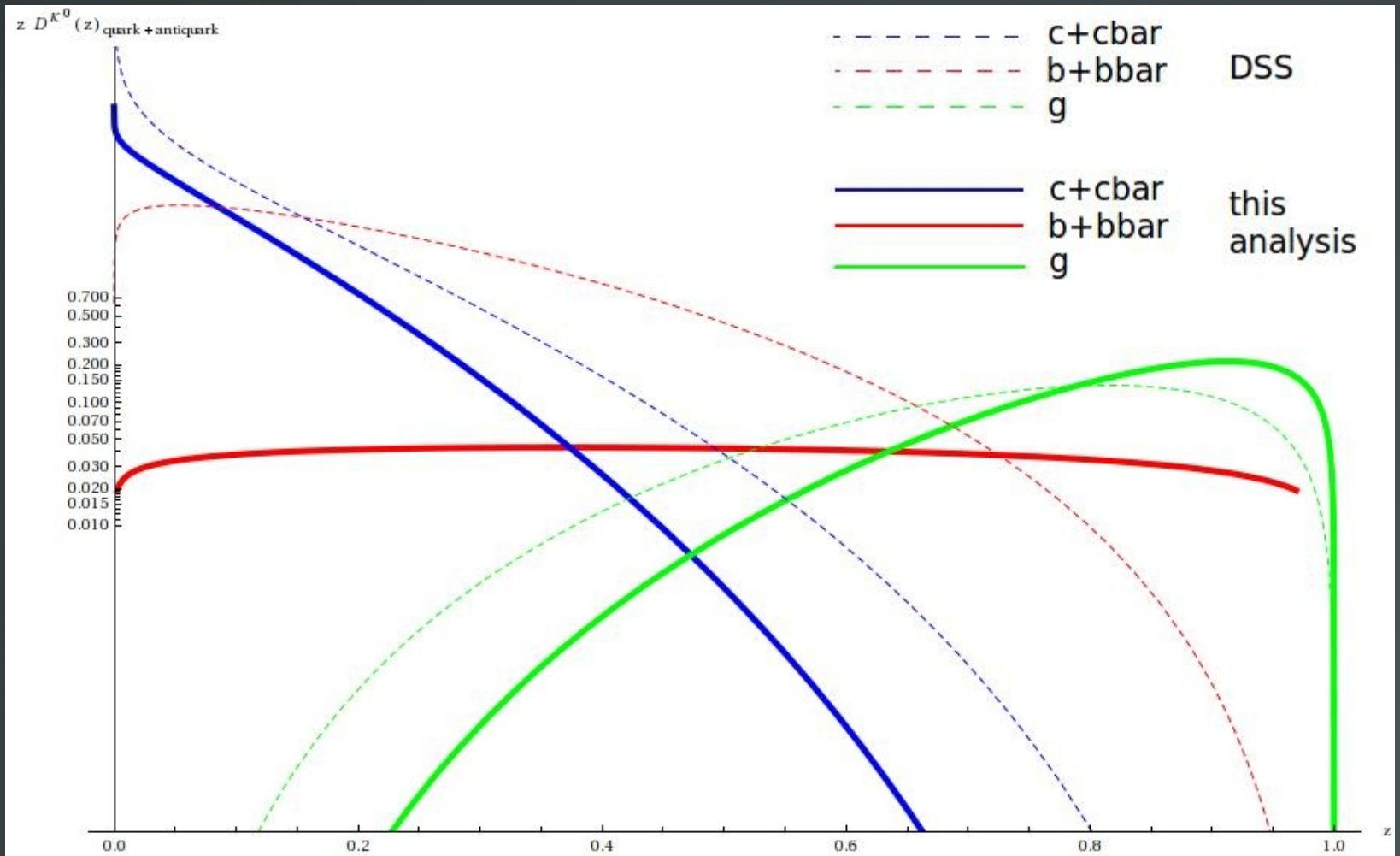
K0 – uds : LO vs. LO by DSS

Results: Kaon-FFs f. light quarks



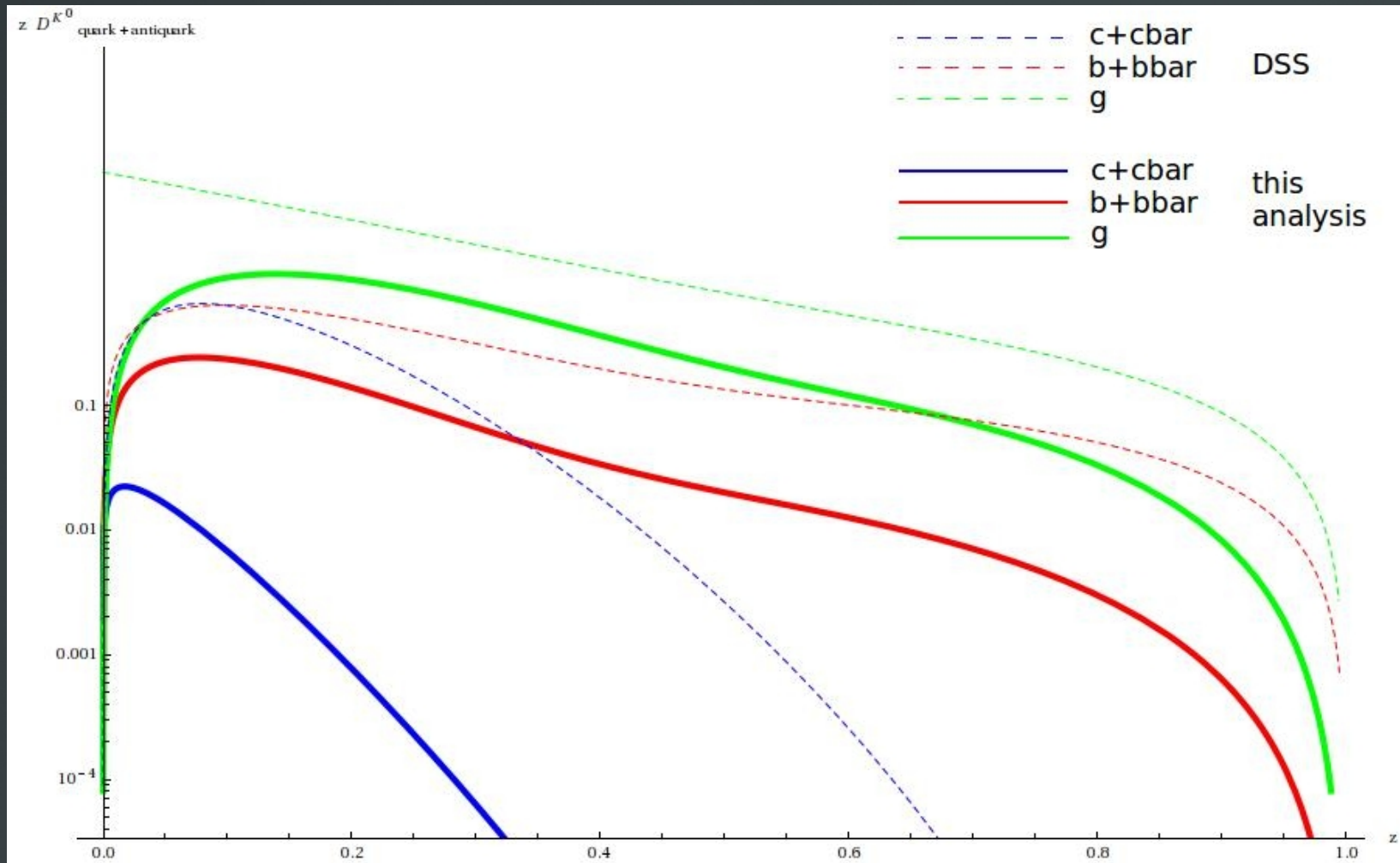
K0 – uds : LO vs. NLO by DSS

Results: Kaon-FFs f. heavy quarks and gluon



K0 – cbg : LO vs. LO by DSS

Results: Kaon-FFs f. heavy quarks and gluon



K0 – cbg : LO vs. NLO by DSS

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Thank you !

