deficiencies of the SM

- does not incorporate gravity
- no mechanism for EWSB
  - Scalar potential put in by hand: \( V(H) = -m_H^2 |H|^2 + \frac{\lambda}{2} |H|^4 \)
- gauge couplings unification – GUT – does not work!
  - cannot accommodate \( \text{SU}(3) \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \) in \( \text{SU}(5) \) or \( \text{SO}(10) \) because coupling strengths do not match at any energy
- no candidate for a dark matter particle
- radiative corrections to the Higgs mass reveals the so-called ‘naturalness’ or ‘hierarchy’ Problem
- Each of these problems has a supersymmetric solution.
  - Does not mean that nature features any/all!
Supersymmetry
Every fundamental matter particle and every force carrier should have a massive "shadow" particle.

This relationship between matter particles and force carriers is called **supersymmetry**.

For example, for every type of quark there may be a type of particle called a "squark."
Supersymmetry ("SUSY")

Sheldon Glashow: “Half of the particles have already been discovered!”

(sorry about the “neutrino” – history sucks)
stop (峙) physics
In the SM the Higgs potential for

\[ V(H) = -m_H^2 |H|^2 + \frac{\lambda}{2} |H|^4 \]

For electroweak symmetry breaking we know that \( v = \sqrt{m_H^2/\lambda} \sim 246 \text{ GeV} \)

Therefore, we have \( 0 < m_H^2 \lesssim (\text{few hundred GeV})^2 \)

and now we know from measurements: \( m_H = 125 \text{ GeV} \)

However, this appears fine-tuned, when we consider the typical size of quantum corrections to \( m_H \)
The correction to the Higgs squared mass parameter from the loop with a Dirac fermion with the Lagrangian $-\lambda_f H \bar{f} f$ is

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{8\pi^2} [\Lambda_{UV}^2 - 3m_f^2 \ln(\Lambda_{UV}/m_f) + \ldots]$$

$\Lambda_{UV}$ denotes the ultraviolet cutoff scale(s) at which new physics enters.

So $m_H^2$ is sensitive to the largest mass scale in the theory.

Let’s assume at the Planck scale $= 10^{19}$ GeV there is new physics, $\Delta m_H^2 \sim 10^{36}$ GeV$^2$ but $m_H^2 \sim 10^4$ GeV$^2$ - 32 orders difference!

$$m_H^2|_{\text{physical}} = m_H^2|_{\text{bare}} - \Delta m_H^2$$

$$10000 = \underbrace{1000000000000000000000000000000010000 - 1000000000000000000000000000000000000}_{10000}$$

Hierarchy or also called Naturalness problem!

H. Eberl
For loop with scalar particle we get also “quadratically divergent” contributions. Using the Lagrangian $-\lambda_S |H|^2 |S|^2$ we get

$$\Delta m^2_H = \frac{\lambda_S}{16\pi^2} \left[ -\Lambda_{\text{UV}}^2 + 2m^2_S \ln(\Lambda_{\text{UV}}/m_S) + \ldots \right].$$

Fermion loops and boson loops give contributions with opposite signs:

$$\Delta m^2_H = -\frac{\lambda_f}{16\pi^2} (-2\Lambda_{\text{UV}}^2) + \ldots \quad \text{(Dirac fermions)}$$

$$\Delta m^2_H = -\frac{\lambda_S}{16\pi^2} \Lambda_{\text{UV}}^2 + \ldots \quad \text{(complex scalar)}$$

For a systematic cancellation of the fermionic by the bosonic contributions we need a symmetry:

SUPERSYMMETRY

H. Eberl
Cancellation of quadratic terms (divergences)

- to avoid quadratic divergences in Higgs mass, otherwise "fine-tuning" would be needed
  - "naturalness"
- sadly, none of the bosons and fermions in the Standard Model can serve as SUSY partners for each other
  - → job for experimentalists
In SUSY:
Particles go in pairs! To each fermion $F$ there is a boson $B$ with the same mass and couplings, and quantum numbers, except the spin, which differs by $\frac{1}{2}$. In the former example

$$\Delta m_{H}^2 = \Delta m_{H}^2|_{\text{fermionic}} + \Delta m_{H}^2|_{\text{bosonic}} = 0$$

with $\lambda_S = \lambda_f^2$ and $m_S = m_f$.

But, no superpartner found up to now, SUSY must be broken! $m_S \neq m_f$

In order to not introduce back the quadratic divergence the dimensionless couplings must be unaffected by the breaking.

Therefore, the breaking of SUSY must be “soft”. This means that it does not change the dimensionless terms in the Lagrangian.

$$\Delta m_{H}^2 = -m_{\text{soft}}^2 \left[ \frac{\lambda}{16\pi^2} \ln(\Lambda_{\text{UV}}/m_{\text{soft}}) + \ldots \right]$$

From $|\Delta m_{H}^2| < m_{H}^2|_{\text{physical}}$ we get $m_{\text{soft}}$ is roughly 1 TeV.

This is the best reason to be optimistic that SUSY will be discovered at the LHC!

H. Eberl
In SUSY:
Particles go in pairs! To each fermion F there is a boson B with the same mass and couplings, and quantum numbers, except the spin, which differs by \( \frac{1}{2} \). In the former example

\[ \Delta m^2_H = \Delta m^2_H |_{\text{fermionic}} + \Delta m^2_H |_{\text{bosonic}} = 0 \]

with \( \lambda_S = \lambda_f^2 \) and \( m_S = m_f \).

But, no superpartner found up to now, SUSY must be broken! \( m_S \neq m_f \)

In order to not introduce back the quadratic divergence the dimensionless couplings must be unaffected by the breaking.

Therefore, the breaking of SUSY must be "soft". This means that it does not change the dimensionless terms in the Lagrangian.

\[ \Delta m^2_H = \Delta m^2_H |_{\text{Higgs}} \]

From \( |\Delta m^2_H| = m_H^2 \beta_H \) we get \( m_S \approx 1 \text{ TeV} \).

This is the best reason to be optimistic that SUSY will be discovered at the LHC!

H. Eberl
Running Coupling constants: Grand Unification

- \( U(1) \) (hypercharge) \( \rightarrow \) \( SU(2)_{\text{left}} \) \( \rightarrow \) \( SU(3) \) (color)
- coupling constants get almost equal at high energies
- but not quite, in Standard Model (left)
- perfect match in Supersymmetry (right)
Dark Matter: galaxy rotation curves
bullet cluster: evidence for Dark Matter

- Colliding galaxy cluster system 1E0657-558 with contours of gravitational mass density (lines) and visible matter density (colors)
massive astrophysical cosmic halo objects?
weakly interacting massive particles?

questions of cosmology to particle physics:
Why is there more matter than anti-matter in the universe?
What is the universe made of? What is dark matter?
What is dark energy?

→ answers to these questions concerning the largest scales
might come from the physics of the smallest scales - elementary
particle physics
how / what are WIMPs?

- heavy
  - otherwise, they would be diluted
  - we see Dark Matter around galaxies
    - ”cold” Dark Matter: ΛCDM
  - so, they cannot be neutrinos!

- neutral

- very weakly interacting
Dark Matter: WIMP searches

- how do you find a WIMP?
  - exclude background $\rightarrow$ go underground
  - watch out for recoils
- WIMP detectors:
  - big dual-phase noble gas volumes
  - crystals
- idea: watch for two different signals
  - ionization
  - scintillation
  - energy deposition ("bolometer")
- or else: make your own WIMP!
  - use a high-energy collider (such as LHC)
  - look for missing transverse momentum
- so far: no WIMPs!
“Axions” were invented to solve the “strong CP problem”
  - why is CP conserved in strong interactions?
they would have to be very light (~meV)
could be “cooled” right after Big Bang by “dynamical friction”
  - after inflation
CAST experiment
looking for solar axions at CERN
The SUSY Algebra

Poincare symmetry:
\[
\begin{align*}
[P^\mu, P^\nu] &= 0 \\
[P^\mu, M^{\rho\sigma}] &= i (\eta^{\mu\rho} P^\sigma - (\rho \leftrightarrow \sigma)) \\
[M^{\mu\nu}, M^{\rho\sigma}] &= i (\eta^{\mu\sigma} M^{\nu\rho} + \eta^{\nu\rho} M^{\mu\sigma} - (\rho \leftrightarrow \sigma))
\end{align*}
\]

SUSY is linked to translations in Minkowski space (because it changes spin) – but:

Coleman-Mandula Theorem:
There is no fusion of internal with space-time symmetries, i.e. the maximal symmetry of the S-Matrix is

\[
\text{Poincare } \boxtimes \text{ Internal symmetries}
\]

unless

Haag-Lopuszanski-Sohnius:
The symmetry algebra is promoted to a Graded Lie Algebra (GLA) with even E and odd O elements closing the GLA

H. Eberl
\[
\begin{align*}
\end{align*}
\]
anticommuting algebra

- in other words, due to these two theorems have to use anticommutators instead of commutators
  - instead of “Lie Algebra” must use “Graded Lie Algebra”
  - “nilpotence” of operators \( \{a,b\}=0 \Rightarrow a\times a = a^2 = 0 \)
- for Standard Model (with “chiral fermions” whose left- and right-handed pieces transform differently - parity violation!):

\[
\begin{align*}
\{Q, Q^\dagger\} &= P^\mu, \\
\{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0, \\
[P^\mu, Q] &= [P^\mu, Q^\dagger] = 0
\end{align*}
\]

- \(P^\mu\): four-momentum generator of spacetime translations
- \(Q\): supersymmetry operator (boson \(\leftrightarrow\) fermion)
The minimal extension is the N=1 SUSY with two Weyl-type spinorial generators $Q, \overline{Q}$ in addition to $P^\mu$ and $M^{\mu\nu}$

$$\{Q_\alpha, \overline{Q}_\dot{\alpha}\} = 2 (\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu, \quad \{Q_\alpha, Q_\beta\} = \{\overline{Q}_\dot{\alpha}, \overline{Q}_\dot{\beta}\} = 0$$

$$[Q, P^\mu] = [\overline{Q}, P^\mu] = 0$$

$$[Q, M^{\mu\nu}] = i\sigma^{\mu\nu} Q$$

Higher SUSYs have more spinorial charges, N > 1

A SUSY transformation turns a bosonic states into a fermionic one and vice versa,

$$Q\ket{\text{Boson}} = \ket{\text{Fermion}} \quad Q\ket{\text{Fermion}} = \ket{\text{Boson}}$$

$Q$ is an anticommuting spinor and therefore a complex object, analogous transformations are performed by $\overline{Q}$.

The single particle states of the theory fall into irreducible representations of this algebra, called supermultiplets. Fermion and boson members of such a multiplet are called superpartners.

Since $P^2$ and $T^a$ commute with $Q$ and $\overline{Q}$, all members of a given supermultiplet must have the same $(mass)^2$ and gauge quantum numbers.

but we have not seen them → sparticles heavier → symmetry “softly” broken
supermultiplets

- combination of massless fermion and complex scalar field: *chiral* or *matter* or *scalar supermultiplet*

- combination of massless spin-1 boson with fermion: *gauge* or *vector supermultiplet*
  - gauge boson $\rightarrow$ gaugino
  - gauginos are left-right symmetric
    - Since the adjoint representation of a gauge group is always its own conjugate, the gaugino fermions must have the same gauge transformation properties for left-handed and for right-handed components.
The SUSY representations – the supermultiplets are

<table>
<thead>
<tr>
<th>Multiplet name</th>
<th>Particle content</th>
<th>Spin content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chiral</td>
<td>$\phi$ $\psi$</td>
<td>0 $\frac{1}{2}$</td>
</tr>
<tr>
<td>Vector</td>
<td>$A_\mu$ $\lambda$</td>
<td>1 $\frac{1}{2}$</td>
</tr>
<tr>
<td>Gravity</td>
<td>$g_{\mu\nu}$ $\psi_\mu$</td>
<td>2 $\frac{3}{2}$</td>
</tr>
</tbody>
</table>

Every multiplet includes a boson and a fermion, their spins differ by $\frac{1}{2}$. As we know, they are connected by SUSY transformations

\[
\text{SUSY} : \quad \phi \leftrightarrow \psi \quad , \quad A_\mu \leftrightarrow \lambda \quad , \quad g_{\mu\nu} \leftrightarrow \psi_\mu
\]

Besides the physical d.o.f. the multiplets also include auxiliary fields $F$ and $D$ to match the number of bosonic and fermionic components also off-shell.
The Minimal Supersymmetric Standard Model

• To every **SM particle** a **SUSY partner** is introduced, both members of the same multiplet and the d.o.f. are more than doubled.

• The structure of the **SM is automatically included**.

• New particles are predicted, super partners (sparticles) of the SM particles – **SUSY models have a rich phenomenology**.

H. Eberl
soft SUSY breaking

- obtain non-supersymmetric physics from supersymmetric theory
  - necessary to reconcile supersymmetry with experiments
  - spontaneous symmetry breaking
  - in supergravity: use slightly modified counterpart of Higgs mechanism
    - gravitinos become massive
- superpartners become much heavier
  - else, their mass would be equal to the mass of the “regular” particles
supergravity

- supergravity (SUGRA) combines the principles of supersymmetry and general relativity
  - searching for “Theory of Everything” (TOE)
- first developed in 1970ies
- mSUGRA (minimal SUper GRAvity)
  - supersymmetry (SUSY) breaking by “super Higgs mechanism” (1982)
- superstring theory (supersymmetric string theory): unlike bosonic string theory, it accounts for both fermions and bosons and incorporates supersymmetry to model gravity
  - “first superstring revolution” (began in 1984)
    - realized that string theory was capable of describing all elementary particles as well as the interactions between them
    - Edward Witten et al.: different superstring theories are limits of an 11-dimensional theory → “M-theory”
We have the SM parameters $\mu + \tan \beta$

**SUSY** terms, defined by:

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right)$$

$$- \left( \tilde{u} a_u \tilde{Q} H_u - \tilde{d} a_d \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_d + \text{c.c.} \right)$$

$$- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u} m_{u u}^2 \tilde{u}^\dagger - \tilde{d} m_{d d}^2 \tilde{d}^\dagger - \tilde{e} m_{e e}^2 \tilde{e}^\dagger$$

$$- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + \text{c.c.}) .$$

$M_1, M_2, M_3$: complex numbers

$a_u, a_d, a_d$: $3 \times 3$ matrices, complex

$m_Q^2, m_L^2, m_{u u}^2, m_{d d}^2, m_{e e}^2$: $3 \times 3$ hermitian matrices

$m_{H_u}^2, m_{H_d}^2$: real numbers, $b$: complex number

105 independent soft breaking parameters, how can this be reduced?

Most of the new parameters imply flavor mixing or $CP$ processes, severe experimental constraints from e.g. rare decays, Kaon physics, B-physics, EDMs of electron, neutron, atoms, DM, …
Parameters of the Standard Model

- $\alpha_1 = (5/3)g'^2 / (4\pi) = 5\alpha / (3 \cos^2 \theta_W)$
- $\alpha_2 = g^2 / (4\pi) = \alpha / \sin^2 \theta_W$
- $\alpha_3 = g_s^2 / (4\pi)$

- $\alpha_1(M_Z) = 0.017$
- $\alpha_2(M_Z) = 0.034$
- $\alpha_3(M_Z) = 0.118 \pm 0.003.$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_e$ Electron mass</td>
<td>511 keV</td>
</tr>
<tr>
<td>$m_\mu$ Muon mass</td>
<td>105.7 MeV</td>
</tr>
<tr>
<td>$m_\tau$ Tau mass</td>
<td>1.78 GeV</td>
</tr>
<tr>
<td>$m_u$ Up quark mass</td>
<td>$\mu_{MS} = 2$ GeV, 1.9 MeV</td>
</tr>
<tr>
<td>$m_d$ Down quark mass</td>
<td>$\mu_{MS} = 2$ GeV, 4.4 MeV</td>
</tr>
<tr>
<td>$m_s$ Strange quark mass</td>
<td>$\mu_{MS} = 2$ GeV, 87 MeV</td>
</tr>
<tr>
<td>$m_c$ Charm quark mass</td>
<td>$\mu_{MS} = mc$, 1.32 GeV</td>
</tr>
<tr>
<td>$m_b$ Bottom quark mass</td>
<td>$\mu_{MS} = mb$, 4.24 GeV</td>
</tr>
<tr>
<td>$m_t$ Top quark mass</td>
<td>On-shell scheme, 172.7 GeV</td>
</tr>
<tr>
<td>$\theta_{12}$ CKM 12-mixing angle</td>
<td>13.1°</td>
</tr>
<tr>
<td>$\theta_{23}$ CKM 23-mixing angle</td>
<td>2.4°</td>
</tr>
<tr>
<td>$\theta_{13}$ CKM 13-mixing angle</td>
<td>0.2°</td>
</tr>
<tr>
<td>$\delta$ CKM CP-violating Phase</td>
<td>0.995</td>
</tr>
<tr>
<td>$g_1$ or $g'$ U(1) gauge coupling</td>
<td>$\mu_{MS} = m_Z$, 0.357</td>
</tr>
<tr>
<td>$g_2$ or $g$ SU(2) gauge coupling</td>
<td>$\mu_{MS} = m_Z$, 0.652</td>
</tr>
<tr>
<td>$g_3$ or $g_s$ SU(3) gauge coupling</td>
<td>$\mu_{MS} = m_Z$, 1.221</td>
</tr>
<tr>
<td>$\theta_{QCD}$ QCD vacuum angle</td>
<td>$\sim 0$</td>
</tr>
<tr>
<td>$v$ Higgs vacuum expectation value</td>
<td>246 GeV</td>
</tr>
<tr>
<td>$m_H$ Higgs mass</td>
<td>$\sim 125$ GeV (tentative)</td>
</tr>
</tbody>
</table>
### Chiral Supermultiplets:

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 0</th>
<th>spin 1/2</th>
<th>SU(3)_C, SU(2)_L, U(1)_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>squarks, quarks</strong> (×3 families)</td>
<td>((\tilde{u}_L) (\tilde{d}_L))</td>
<td>((u_L) (d_L))</td>
<td>(3, 2, 1/6)</td>
</tr>
<tr>
<td>(u)</td>
<td>(\tilde{u}^*_R)</td>
<td>(u^\dagger_R)</td>
<td>(3, 1, -2/3)</td>
</tr>
<tr>
<td>(\bar{d})</td>
<td>(\tilde{d}^*_R)</td>
<td>(d^\dagger_R)</td>
<td>(3, 1, 1/3)</td>
</tr>
<tr>
<td><strong>sleptons, leptons</strong> (×3 families)</td>
<td>((\tilde{\nu}) (\tilde{e}_L))</td>
<td>((\nu) (e_L))</td>
<td>(1, 2, -1/2)</td>
</tr>
<tr>
<td>(\bar{e})</td>
<td>(\tilde{e}^*_R)</td>
<td>(e^\dagger_R)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td><strong>Higgs, higgsinos</strong></td>
<td>(H_u)</td>
<td>(H^+_u) (H^0_u)</td>
<td>(1, 2, +1/2)</td>
</tr>
<tr>
<td>(H_d)</td>
<td></td>
<td>(\tilde{H}^+_u) (\tilde{H}^0_u)</td>
<td></td>
</tr>
</tbody>
</table>

**Y=Q-T\(_3\)**

- **weak hypercharge:** electric charge minus 3\(^{rd}\) component of weak isospin
- **alternative definition:**
  \[Y=2 \times Q - T\(_3\)\]

**Gaugino Supermultiplets:**

<table>
<thead>
<tr>
<th>Names</th>
<th>(B^+)</th>
<th>(B^0)</th>
<th>(1, 1, 0)</th>
</tr>
</thead>
</table>

**Gluino, gluon**

**Winos, W bosons**

**Bino, B boson**
### Chiral supermultiplets:

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 0</th>
<th>spin 1/2</th>
<th>$SU(3)_C, SU(2)_L, U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>squarks, quarks</td>
<td>$Q$</td>
<td>$(\widetilde{u}_L , \widetilde{d}_L)$</td>
<td>$(u_L , d_L)$</td>
</tr>
<tr>
<td>$(\times 3$ families)</td>
<td>$\bar{u}$</td>
<td>$\widetilde{u}_R^*$</td>
<td>$u_R^\dagger$</td>
</tr>
<tr>
<td></td>
<td>$\bar{d}$</td>
<td>$\widetilde{d}_R^*$</td>
<td>$d_R^\dagger$</td>
</tr>
<tr>
<td>sleptons, leptons</td>
<td>$L$</td>
<td>$(\tilde{\nu} , e_L)$</td>
<td>$(\nu , e_L)$</td>
</tr>
<tr>
<td>$(\times 3$ families)</td>
<td>$\bar{e}$</td>
<td>$\widetilde{e}_R^*$</td>
<td>$e_R^\dagger$</td>
</tr>
<tr>
<td>Higgs, higgsinos</td>
<td>$H_u$</td>
<td>$(H_u^+ , H_u^0)$</td>
<td>$(\tilde{H}_u^+ , \tilde{H}_u^0)$</td>
</tr>
<tr>
<td></td>
<td>$H_d$</td>
<td>$(H_d^0 , H_d^-)$</td>
<td>$(\tilde{H}_d^0 , \tilde{H}_d^-)$</td>
</tr>
</tbody>
</table>

### Gauge supermultiplets:

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 1/2</th>
<th>spin 1</th>
<th>$SU(3)_C, SU(2)_L, U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluino, gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>$(8, 1, 0)$</td>
</tr>
<tr>
<td>winos, W bosons</td>
<td>$\tilde{W}^\pm$</td>
<td>$W^\pm , W^0$</td>
<td>$(1, 3, 0)$</td>
</tr>
<tr>
<td>bino, B boson</td>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td>$(1, 1, 0)$</td>
</tr>
</tbody>
</table>
R-parity

- baryon and lepton numbers are conserved
  - at least to very good approximation
  - proton lifetime > $10^{33}..10^{34}$ years
  - must be reproduced by SUSY

- R-parity:
  \[ P_R = (-1)^{3(B-L)+2s} \]
  - all Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$)
  - all squarks, sleptons, gauginos, and higgsinos have odd R-parity ($P_R = -1$)

- Lightest Supersymmetric Particle with ($P_R = -1$) must be stable!
  - but we have not seen any!
  - \( \Rightarrow \) “LSP” as dark-matter candidate
  - typically assumed to be (mixture of) electroweak gauginos and Higgsinos
  - called “neutralino”
  - or sneutrino or gravitino
co-annihilation of LSPs

- Have to match observed Dark-Matter density with measured cross sections!
- As the universe cooled and expanded after the Big Bang, the heavier sparticles could no longer be produced, and they eventually annihilated or decayed into neutralino LSPs.
- Some of the LSPs pair-annihilated into final states not containing sparticles.
- If there are other sparticles that are only slightly heavier, then they existed in thermal equilibrium in comparable numbers to the LSP, and their co-annihilations are also important in determining the resulting dark matter density.
co-annihilation of LSPs

Figure 10.13: Contributions to the annihilation cross-section for neutralino dark matter LSPs from (a) $t$-channel slepton and squark exchange, (b) near-resonant annihilation through a Higgs boson ($s$-wave for $A^0$, and $p$-wave for $h^0$, $H^0$), and (c) $t$-channel chargino exchange.

Figure 10.14: Some contributions to the co-annihilation of dark matter $\tilde{N}_1$ LSPs with slightly heavier $\tilde{N}_2$ and $\tilde{C}_1$. All three diagrams are particularly important if the LSP is higgsino-like, and the last two diagrams are important if the LSP is wino-like.

Figure 10.15: Some contributions to the co-annihilation of dark matter $\tilde{N}_1$ LSPs with slightly heavier sfermions, which in popular models are most plausibly staus (or perhaps top squarks).
what can we hope to see?

- SUSY could appear as correction (contribution) to processes
  - e.g., $\mu \rightarrow e\gamma$, $b \rightarrow s\gamma$, neutral meson mixing, electric dipole moments for the neutron and the electron
    - measurements at the “precision frontier”
  - but hard to prove that effect is due to SUSY and not something else

- at LHC, we are limited by collision energy
  - $13 \ldots 14$ TeV
    - measurements at the “energy frontier”
  - but colliding partons (quarks or gluons) have only part of that energy!
    - (lepton collider would be nicer but cannot reach the energy)
  - $\rightarrow$ not much hope to see particles of mass $>> 1$ TeV

- so, which are the lightest “new” (s)particles?
neutralinos and charginos

- higgsinos and electroweak gauginos mix because of electroweak symmetry breaking:
  \[ m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4} \]
  \[ m_{\tilde{C}_1} < m_{\tilde{C}_2} \]

- lightest neutralino \( \tilde{N}_1 \) usually assumed to be the LSP ("lightest supersymmetric particle")
  - unless there is a lighter gravitino
  - or unless R-parity is not conserved
  - only MSSM particle that can make a good dark matter candidate
• color octet fermion $\rightarrow$ cannot mix with other particle
• in certain models (MSUGRA):
  gluino mass : bino mass : wino mass scale as
  \[ M_3 : M_2 : M_1 \approx 6 : 2 : 1 \]
• $\rightarrow$ gluino is (could be) considerably heavier than the lighter neutralinos and charginos
in principle, any scalars with the same electric charge, R-parity, and color quantum numbers can mix with each other

but there are good reasons to believe mixing angles are small
- at least for first two families
- hypothesis of flavor-blind soft parameters

third family (top, bottom, tau): strong Yukawa couplings can introduce big corrections
- Yukawa coupling: coupling to Higgs field (high mass of top!)
- can introduce significant splitting between the 2 stop states $\tilde{t}_1, \tilde{t}_2$
  (mixtures of $\tilde{t}_L, \tilde{t}_R$)
- corrections can reduce mass

→ third family sparticles should (could) be much lighter than 1st and 2nd family sparticles
in MSSM (“Minimum Supersymmetric Model”) there are 33 distinct masses corresponding to undiscovered particles:

<table>
<thead>
<tr>
<th>Names</th>
<th>Spin</th>
<th>$P_R$</th>
<th>Gauge Eigenstates</th>
<th>Mass Eigenstates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs bosons</td>
<td>0</td>
<td>+1</td>
<td>$H^0_u , H^0_d , H^+_u , H^-_d$</td>
<td>$h^0 , H^0 , A^0 , H^\pm$</td>
</tr>
<tr>
<td>squarks</td>
<td>0</td>
<td>−1</td>
<td>$\tilde{u}_L , \tilde{u}_R , \tilde{d}_L , \tilde{d}_R$</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{s}_L , \tilde{s}_R , \tilde{c}_L , \tilde{c}_R$</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{t}_L , \tilde{t}_R , \tilde{b}_L , \tilde{b}_R$</td>
<td>$\tilde{t}_1 , \tilde{t}_2 , \tilde{b}_1 , \tilde{b}_2$</td>
</tr>
<tr>
<td>sleptons</td>
<td>0</td>
<td>−1</td>
<td>$\tilde{e}_L , \tilde{e}_R , \tilde{\nu}_e$</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\mu}_L , \tilde{\mu}<em>R , \tilde{\nu}</em>\mu$</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\tilde{\tau}_L , \tilde{\tau}<em>R , \tilde{\nu}</em>\tau$</td>
<td>$\tilde{\tau}_1 , \tilde{\tau}<em>2 , \tilde{\nu}</em>\tau$</td>
</tr>
<tr>
<td>neutralinos</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{B}^0 , \tilde{W}^0 , \tilde{H}^0_u , \tilde{H}^0_d$</td>
<td>$\tilde{N}_1 , \tilde{N}_2 , \tilde{N}_3 , \tilde{N}_4$</td>
</tr>
<tr>
<td>charginos</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{W}^\pm , \tilde{H}^+_u , \tilde{H}^-_d$</td>
<td>$\tilde{C}^\pm_1 , \tilde{C}^\pm_2$</td>
</tr>
<tr>
<td>gluino</td>
<td>1/2</td>
<td>−1</td>
<td>$\tilde{g}$</td>
<td>(same)</td>
</tr>
<tr>
<td>goldstino (gravitino)</td>
<td>1/2 (3/2)</td>
<td>−1</td>
<td>$\tilde{G}$</td>
<td>(same)</td>
</tr>
</tbody>
</table>
Renormalization group evolution of scalar and gaugino mass parameters in the MSSM with MSUGRA boundary conditions
possible mass scales (just one scenario)

"Mass scales deliberately omitted.

These spectra are presented for entertainment purposes only!

No warranty, expressed or implied, guarantees that they look anything like the real world."

source:
A Supersymmetry Primer
Stephen P. Martin
hep-ph/9709356, version 6, September 2011
experimental signals for supersymmetry

- “So far, the experimental study of supersymmetry has unfortunately been confined to setting limits.”
  - still true today

- sparticle production at colliders:
  - electroweak (dominant at lower energies / Tevatron):
    \[
    \begin{align*}
    q\bar{q} & \rightarrow \tilde{C}_i^+\tilde{C}_j^-, \tilde{N}_i\tilde{N}_j, & u\bar{d} & \rightarrow \tilde{C}_i^+\tilde{N}_j, & d\bar{u} & \rightarrow \tilde{C}_i^-\tilde{N}_j, \\
    q\bar{q} & \rightarrow \tilde{\ell}_i^+\tilde{\ell}_j^-, \tilde{\nu}_\ell\tilde{\nu}_\ell^*, & u\bar{d} & \rightarrow \tilde{\ell}_L^+\tilde{\nu}_\ell & d\bar{u} & \rightarrow \tilde{\ell}_L^-\tilde{\nu}_\ell^*.
    \end{align*}
    \tag{10.1.1}
    \tag{10.1.2}
    \]

  - QCD (dominant at LHC energies):
    \[
    \begin{align*}
    gg & \rightarrow \tilde{g}\tilde{g}, \tilde{q}_i\tilde{q}^*_j, & \\
    gq & \rightarrow \tilde{g}\tilde{q}_i, & \\
    q\bar{q} & \rightarrow \tilde{g}\tilde{g}, \tilde{q}_i\tilde{q}^*_j, & \\
    q\bar{q} & \rightarrow \tilde{q}_i\tilde{q}_j.
    \end{align*}
    \tag{10.1.3}
    \tag{10.1.4}
    \tag{10.1.5}
    \tag{10.1.6}
    \]
Figure 10.1: Feynman diagrams for electroweak production of sparticles at hadron colliders from quark-antiquark annihilation. The charginos and neutralinos in the $t$-channel diagrams only couple because of their gaugino content, for massless initial-state quarks, and so are drawn as wavy lines superimposed on solid.
production by gluon-gluon and gluon-quark fusion

Figure 10.2: Feynman diagrams for gluino and squark production at hadron colliders from gluon-gluon and gluon-quark fusion.
production by quark-(anti)quark interaction

Figure 10.3: Feynman diagrams for gluino and squark production at hadron colliders from strong quark-antiquark annihilation and quark-quark scattering.
signatures

- if R-parity: LSP escapes detector
  - \( \rightarrow \) missing \( p_t \) (“ET\(_{\text{miss}}\)”, “MET”)
    - plus jets and/or leptons

- important backgrounds:
  - W+jets
    - with the W decaying to lepton-\( \bar{\nu} \) (when charged lepton is missed)
      - reduce by cut on transverse mass
  - Z+jets, with \( Z \rightarrow \nu\bar{\nu} \)
  - \( t\bar{t} \) production
    - with \( W \rightarrow l\nu \), when the charged lepton is missed
Supersymmetric ("SUSY") particles could show very clear signatures due to cascade decays but none have been found so far!
if we don’t see anything: exclusion plots

- shading: experimental limit
- lines: intersection with theory predictions

- $m_{\text{LSP}} = 3$
- $m_{\text{LSP}} = 2$
- $m_{\text{LSP}} = 1$

CMS Preliminary, 19.5 fb$^{-1}$, $\sqrt{s} = 8$ TeV

$pp \rightarrow \tilde{g}\tilde{g}, \tilde{g} \rightarrow q\bar{q}\tilde{\chi}_1^0$ NLO+NLL exclusion

- Observed $\pm 1\sigma_{\text{theory}}$
- Expected $\pm 1\sigma_{\text{experiment}}$

95% C.L. upper limit on cross section (pb)
How is SUSY doing?

- “If supersymmetry is the solution to the hierarchy problem, then the LHC should be able to establish strong evidence for it”
  - (S. Martin, 2011)

- it is getting increasingly unlikely to find something that does all the main three jobs SUSY was expected to accomplish for a long time:
  - fine-tuning / naturalness
  - grand unification
  - dark matter

- but there might still be something out there that matches two of these ideas

- to really “kill SUSY”, much higher energies would be needed than available at present-day colliders
Save Susy! ... experiment

- much of the parameter space in simplest models excluded
  - lower mass limits set by LHC
- look in regions difficult to investigate (and therefore not screened so far):
  - “compressed spectra”: small mass gap between NLSP and LSP \( \rightarrow \) soft visible decay products
    - “(next to) lightest supersymmetric particle”
  - “long-lived particles”: decay vertices displaced from beam interaction point
    - not in trigger so far
Save Susy! … theory

- “split SUSY”: sacrifice naturalness
  - accept fine-tuning

- anthropic principle:
  - in a less favorable universe we would not be there to ask these questions
  - was not “politically correct” for some time ... but times change

- split SUSY: very heavy gluino
  - no good to solve “hierarchy” or “naturalness” problem
  - but still achieves gauge coupling unification and has a Dark-Matter candidate